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Teachers' Mathematical Pedagogical Content Knowledge: Some Trends in Search of Adequate Knowledge for Effective Teaching

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ABSTRACT In this paper, as a case study, the teachers' Pedagogical Content Knowledge (PCK) was determined by analyzing how teachers taught a topic on mathematics and their responses to the questions on the same topic. Data was collected using face-to-face interview schedules with short-answer and multiple-choice type items. A one sample t-test was used to analyze the collected data. The items that were open ended were compiled to generate quantitative data which were then subjected to quantitative analysis. The findings showed that many teachers had no adequate PCK to understand well the characteristics of the problems tackled, solutions obtained by different methods, their interpretation and lack of understanding of students' problems in learning. It was concluded that teachers needed improvement in their mathematical PCK for effective teaching of the subject.

INTRODUCTION

The debate on Pedagogical Content Knowledge (PCK) required by mathematics teachers to teach the subject effectively, has long existed and continues in the 21st century. The mastery by teachers of PCK has always been related to the quality of teachers in mathematics instruction and performance of students in the subject as it enables teachers to make knowledge accessible and understandable to students. Shulman (1986) was the first to describe the type of knowledge required by teachers, as the knowledge of understanding students' ways of thinking; the teacher's ability to diagnose sources of students' errors; and knowledge of various alternative ways of representing specific topics (Shulman 1986; Carlsen 1987; Tirosh et al. 1998). Pedagogical Content Knowledge was also described as the knowledge of what made the learning of specific topics easy or difficult and as the knowledge that comprised of subject matter knowledge (SMK) and pedagogical knowledge (PK) (Shulman 1987). The PK was described as the general knowledge that the teacher acquired in a methods or professional studies course. Pedagogical knowledge consists of the learner development theories, learning theories, classroom management, and assessment procedures. Shulman (1986) viewed teacher education from the concept of subject matter and pedagogy where the two combined had to effectively prepare teachers. Since Shulman's typology of PCK, a number of researchers have described PCK in various ways, critiqued and developed the typology.

Veal and Maknister (1999) described PCK as a way of describing the knowledge possessed by the expert teachers and Tirosh (1999) described it as the method of teaching for understanding. Other researchers such as Grossman (1990), Zeidler et al. (2002), Banks et al. (2005), Wilson and Demetriou (2007) have described PCK as the content knowledge (the knowledge of the subject content that needed to be taught), general pedagogical knowledge (knowledge of different teaching strategies, classroom management strategies, assessment strategies etc), context knowledge (knowing about the background of the learners, knowing the organizational culture of the school etc), and pedagogical content knowledge. Despite above critiques and developments of Shulman's (1986) PCK typology, there are still debates on the type of PCK needed by teachers for quality instruction.

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In mathematics teachings, a number of studies have described PCK in various ways. Some mathematics educators have focused on what is termed as Specialized Content Knowledge (SCK) or content knowledge for teaching, Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC) (Blomeke and Delaney 2012). The debate continues in search of mathematical knowledge required by mathematics teachers. Recently, teacher education research and research on practicing teachers have focused more on the knowledge base of teachers' classroom practice (Blomeke and Delaney 2012), the main focus of this paper.

Theoretical Framework

This study adopted Ma's (1999), Lienhart's and Smith's (1985) and Ball (1990) framework components that suited to achieve the study objectives. Ma's (1999) framework component used in this study required the teacher to (1) explain how he/she taught a particular selected concept; (2) explain how to perform a particular mathematical task; and (3) explain scenarios where a student comes up with a different way to solve the problem that led to a seemingly correct solution process but misrepresented the correct conception(s). The authors selected Ma's (1999) framework because the 1st category required the teacher not only to show the ability of content knowledge that a teacher possessed but also how the teacher was able to teach that content. This assertion was in line with the authors' beliefs that teachers who possessed enough content knowledge should be able to teach the content thoroughly. The second and third categories wanted to know how the teacher explained the solution of the problem obtained and be able to explain different types of solutions that might be provided by students and sometimes provided contradicting results.

In the second framework, the authors adopted Lienhart and Smith's (1985) framework components that required the teacher to explain particular mathematical terms, compare and explain the meaning of particular terms, draw and discuss the meaning of the sketch, and to explain problems that students faced in understanding certain concepts when teaching. This framework was also preferred by the authors because it did not only test the teacher's knowledge of terms but also the comparison of the terms and explanation of their meaning. It also required teachers to draw a graph to give them the opportunity to show their psychomotor skills they could apply during classroom instruction. The discussion of the sketches would indicate higher abilities of the teacher. And the requirement for the teacher to explain why some students found difficulties in learning certain concepts showed how the teacher was able to understand student problems in learning.

The third framework by Ball (1990) used structured multiple-choice items that were posed using various scenarios related to situations that teachers faced in the classroom. The authors chose this framework and combined it with both Ma's (1999) and Lienhart and Smith's (1985) frameworks above to construct multiple-choice items. The authors chose to use multiple-choice items because they covered most areas of the topic considered in the study and were able to assess the general teacher's abilities to teach the topic. The multiple-choice items used also tested higher cognitive abilities of the teacher.

The above three frameworks used in this study by Lienhart and Smith (1985), Ball (1990), and Ma (1999) involved short and long essay type questions that covered a wider scope of the topic items and thus advantageous to test teacher knowledge. Questions involving terms such as *state and explain* were short essay type of questions that tested lower levels of knowledge. Questions involving creation of mathematical scenarios, comparison of mathematical terms and discussion thereof tested the teacher's higher cognitive abilities. The framework of the components used in this study covered most cognitive skill areas required by the teacher to teach the topic effectively.

The Nature and Scope of the Questions Asked to Teachers

In this study, the teachers' knowledge of linear systems that was determined involved knowledge on the characteristics of the linear equations with two variables and the knowledge on the type of linear systems that had a solution or no solution. The questions asked sought to know whether teachers had knowledge of the characteristics of systems of linear equations with two variables and their interpretations. The linear systems given were

$\{ x+3y=8; 2x+6y=16 \}$ and $\{ x-y=10; x-y=3 \}$

The teachers were given scenarios of different solutions that might be given by students and were required to explain how the students could have opted for particular solutions and provided different answers. The scenarios required the teacher to observe the equation coefficients of the system and find out whether the matrix was singular or non-singular. Identification of this was to show that the teacher was able to identify simple linear systems that had solutions (non-singular) or inconsistent systems (singular) that had no solutions as taught at the school level. Knowledge of above was important to determine whether the teacher was able to identify linear systems that involved practical problems in everyday life. In real life situations, most practical problems involve linear systems. Furthermore, teachers were required to explain the difficulties that learners faced when linear equations topic was taught.

Objectives of the Study and Expectation

The study sought to explore the teachers' knowledge in mathematics teaching and the ways teachers handled particular students' responses in answering questions posed by teachers teaching at grade 12 secondary school level. The study was an attempt to know the type of dialogue teachers had and their reactions to different scenarios of students' solutions and how teachers understood difficulties faced by students in mathematics learning. The topic considered was a system of linear equations with two variables. Teachers were required to explain the characteristics of given equation systems, explain solutions obtained by different methods and to also explain students' problems in understanding the concepts in linear equations.

Research Questions

1. What was the teachers' pedagogical content knowledge in teaching linear equations of two variables?
2. What were the responses of teachers on questions concerning the characteristics of linear equations of two variables?
3. Did teachers have an understanding of the students' problems on linear equations?

METHOD

Design

The study was mainly quantitative and used face-to-face interview schedules to collect data from mathematics teachers. The qualitative part in the interviews involved information from teachers to respond to open ended questions. Even though interviews are associated with qualitative research designs, the authors resorted to using a face-to-face interview rather than asking teachers to sit for a test because teachers were reluctant to do the latter.

Population/ Sampling

The target population was teachers who taught mathematics at senior secondary school level. The sample involved 40 Grade 12 mathematics teachers who were randomly selected from 10 targeted schools in a certain district. The random sampling procedure to get 40 from the 70 teachers in the district used a developed frame of reference where all the 70 teachers were assigned numbers from 1 to 70. Each of the 70 teachers was assigned a number between 1 and 70. We then used a table of random numbers to select the first 40 random numbers. These were numbers in the range 1-70 (with 1 and 70 included). Any teacher whose assigned number matched was included in the study.

Validity and Reliability of Instrument

The validity of the instruments was determined to ensure that they measured the intended content area for respondents at the levels they taught. Constructed questions on the topic were matched to the mathematics syllabus used to ensure that knowledge taught in the classroom was well captured in the interviews. The reliability was determined through Cronbach's alpha and the Split-half method. The Cronbach's alpha coefficient was 0.7 and this value was sufficient enough for consistency of an instrument (Creswell 2003).

Data Analysis

A one sample t-test was used to analyze the collected data. The interview items that were open ended were compiled to generate quantitative data which were then subjected to quantitative analysis. In the analysis, frequencies were used to describe the distribution of teacher performance on interview items. In particular, frequency tables and percentages were used to present and describe the performance respectively. The one sample t-test was used to measure the extent of how the performance of teachers varied. The one sample t-test was considered proper to use as the study dealt with one sample of teachers. Results from the t-test were first presented in tables followed by a systematic descriptive and interpretive analysis.

FINDINGS

The teachers' responses obtained from questions that centered on a given system of linear equations indicated that many teachers failed to explain two scenarios that gave different solutions of the first system of two linear equations. That is, 0 = 0, and 16 = 16. Teachers were required to show how they would explain such a situation to learners who provided two different answers from one system: . The teachers had to explain the physical nature of such a system that resulted into two solutions which were provided by the students. The first solution (0 = 0) was obtained by the student using the elimination method and the second solution (16 = 16) was obtained by substitution method. This system is basically not an independent system as the two equations are the same (when the second equation is divided by 2 on both side we get the first equation) meaning that the system has a singular matrix. Such systems of linear equations have many infinite solutions not suitable in many practical problems and have to be avoided. Fifty five percent (55%) of the teachers indicated that such solutions were possible for the level of students taught (Table 1) and about 43% said the system had no solution (Table 2) while others were not sure.

The finding in Table 2 showed that 48% of the teachers indicated the system had a solution which meant that teachers could not explain to learners the nature of such a system that was singular.

Further questions were then asked to explore knowledge of the teacher focusing on the characteristics of equation systems. The equation system was engaged to study teachers' knowledge on equation intercept on the x and y axes, gradients of the lines, nature of parallel and coinciding lines, and coefficient matrix of the linear equations were posed to teachers to understand how they could explain these to the students. Teachers were able to explain well equation intercept on the x and y axes (83%), gradients of the lines (78%) but performed poorly on knowledge concerning consistency and independence of the system (30%), knowledge on coefficient of the matrix of the system (48%), knowledge on parallel lines (30%) and coinciding/intersection of lines (34%). The performance of teachers is presented in Table 3.

Teachers were also asked to explain what they thought made learning of linear systems difficult to their students. About 67.5% did not know

Table 1: Teachers' performance on PCK question 1(a)

		Frequency	*Percent*	*Valid percent*	*Cumulative percent*
Valid	Yes	22	55.0	55.0	55.0
	No	14	35.0	35.0	90.0
	I am not sure	4	10.0	10.0	100.0
	Total	40	100.0	100.0	
	Total	40	100.0		

Table 2: Teachers' performance on PCK question 1(b)

		Frequency	*Percent*	*Valid percent*	*Cumulative percent*
Valid	Yes	17	42.5	42.5	42.5
	No	19	47.5	47.5	90.0
	I am not sure	4	10.0	10.0	100.0
	Total	40	100.0	100.0	
	Total	40	100.0		

what made linear equations difficult to their students and 32.5% were able to explain.

Table 3: Teachers' performance on question 2

Item knowledge (on PCK)	*Score in %*
Equation intercept on the x and y axes	83
Gradients of the lines	78
Consistency and independence of the system	30
Coefficient of the matrix of the system	48
Parallel lines	30
Coinciding/intersection of lines	34

General Performance of Teachers in PCK

The mean performance of teachers in PCK was 52.38% with standard deviation of 18.54 and the standard error of 2.93 (Table 4). The large size of the standard deviation (18.54) indicated that individual scores were scattered around the mean. Computed value of t (/t/ =14.544) was greater than the critical value (t = 2.021), for alpha (a) = 0.05 significance level with 39 degrees of freedom and the p-value = 0.00 was less than the alpha value chosen for the test.

Table 4: Results of the one sample statistics

	N	*Mean*	*Std. deviation*	*Std. error mean*
PCK Total	40	52.38	18.54	2.93

The results of the one sample t-test lied within the region of rejection chosen for the test. The evidence from the results of the one sample t-test revealed that it existed a statistically significant difference between the sample mean (52.38%) and the test value (95) (Table 5). Since the results pointed towards a statistically significant difference between the means, it was concluded that the sample mean of 52.38% was different from a test value of 95%. Consequently, conclusion based on the results of the one sample t-test was that teachers in the sample possessed little PCK of linear systems.

Table 5: Results of the one sample t-test for PCK

	Test value = 95					
	t	*df*	*Sig. (2-tailed)*	*Mean difference*	*95% Confidence*	
					Lower	*Upper*
PCKTOTAL	-14.544	39	.000	-42.63	-48.55	-36.70

DISCUSSION

The failure of teachers to explain thoroughly the characteristics of the equation systems given was an indication that teachers lacked PCK in teaching linear equations at school level. The failure to know the characteristics of the equations to identify whether the equations had solutions or not further showed that teachers lacked understanding of the nature of the problems which were solved and the meaning thereof. Knowledge of the equations characteristics was essential in the interpretation of the equation systems in real life problems to ensure more understanding of the subject by student. Teachers seemed to know which skills were needed to solve the equation systems using available methods in an attempt to obtain the answers, having the notion that any method applied had to provide the correct answer. The solutions in the examples were solvable by applying two methods which yielded two different solutions that teachers were convinced were correct and they became defensive of their solutions. Teachers arrived at the conclusion that the system of the equations had solutions without having investigated the nature of the systems. Identification of the nature of equations was an essential step for teachers to know systems that had a practical application in real life problems. In the problems posed to teachers, one of the equations was a factor of the other equation, meaning that the two equations were actually the same. The best way to understand the nature was to check the system coefficient matrix and find the determinant of the system. If the determinant was zero then it meant the system was singular and such a system would not be preferred for practical problems. It seemed teachers were used to solving no-singular systems that were directly solved to eventually obtain the answers.

In this study, teachers performed well in questions that asked to determine the intercepts of the two equations and gradients of the lines. Experience has shown that many teachers in schools emphasized knowledge on the sketching of graphs, determination of the gradients of equations, and the x and y intercepts without explaining the nature of the graphs and their meaning. Teachers also provided students with problems in linear equations that had solutions and which were solved by known methods such as graphical method (to determine where the two equations intercepted), elimination and substitution. In these cases, teachers were determined to obtain the solutions using these methods, meaning that the equations involved were to be non-singular. It was important for teachers to understand the nature of equations being solved so as to know the type of practical problems that involved linear systems with non singular matrices.

The ability of teachers to solve the problems using either method of the solution was an indication that teachers were knowledgeable in the topic considered, most likely acquired the Subject Matter Knowledge (SMK) from training at teacher training institutions. The knowledge that the teachers displayed in this study can be described as SMK in the topic possessed by the 'expert' teacher (Veal and Maknister 1999) but not the knowledge for the learner's understanding (Tirosh 1999). The teachers' knowledge could also be described as the lack of knowledge for teaching (Blomeke and Delaney 2012).

Many teachers in this study were not able to understand how students could think in different ways to produce different solutions of the same problem which was given. As argued by Shulman (1986), teachers' PCK should involve knowledge of understanding how student think and the ability of the teacher to diagnose sources of students' errors. Teachers should have shown the knowledge that solving the problems given to them by using different methods would have resulted into two different solutions, indicating that the equation system was inconsistent. The importance of the teachers' understanding of the learners' ways of representing specific topics was described by (Shulman 1986; Carlsen 1987; Tirosh et al. 1998) as essential in mastery of PCK. The teachers' failure to explain the students' problems in learning the topics was an indication that teachers lacked the knowledge about the background of the learners' abilities in the topic, also an important aspect in teachers' mastery of PCK.

The development of students' interest in the topic could be raised if students were taught the equations with an application approach. However, it remained doubtful whether the teachers acquired enough knowledge at training institutions to relate the content knowledge to real life problems. The nature of teacher preparation at training institutions in relation to real life aspects has been an issue of debate in curriculum designs and instruction. It remains doubtful whether the scenarios provided in this study were part of teacher preparation. Generally, an average teachers' performance of 52% in PCK indicated that about 48% of teachers had no adequate knowledge to teach linear equations at the considered level of schooling.

Teachers in this study had an average of nine (09) year experience of teaching and it could be assumed that they had acquired substantial PCK for effective teaching. Some researchers had argued that work-related knowledge (Billett 2004; Evans and Rainbird 2002; Fuller and Unwin 2004) and extensive life experiences to refer to in their classroom practice (Merriam and Clark 1993; Jarvis 2005) contributed to improved teachers' PCK. These authors argue that teachers' PCK may not be acquired at teacher training, but rather developed on the field as the teacher gained more experience in teaching resulting from the interaction of SMK and PK. While teachers' SMK and PK were necessary at teacher training to prepare teachers (Shulman 1986), the development of the combination of the two was necessary in professional development of teachers in the field. Preparation of teachers at training institutions may not be enough for teachers' adequate PCK and thus the need for professional development of teachers. While the teachers' experience in the field could play a great role in improving the teachers' PCK, continuous professional development of teachers in the field is essential. This study could not establish whether the teachers' experience of nine (09) years had helped them to improve in PCK since the PCK knowledge of their entry points to the field was not known to the researchers. Bertram (2011) argues that knowledge-in-practice of teachers was to be developed through participation and by practicing new teaching or assessment strategies in the presence of a supportive colleague in the school situation. Bertram's strategy could be strengthened if experts in the subjects (from training institutions) can be part of the team that supports teachers in schools.

In this study, the teachers' PCK can be described as teachers' lack of Mathematical Knowledge for Teaching (MKT) that comprised of knowledge of the topic content, ways of teaching that content, understanding of students' background of their knowledge in the topic taught and teaching strategies to teach that had an application approach.

CONCLUSION

The low performance of teachers in PCK led to the conclusion that teachers who participated in the study had no thorough knowledge needed to teach effectively linear equations at grade 12 level. Possession of SMK in linear equations without knowledge to explain its characteristics to students was a testimony that teachers lacked adequate PCK. There were a number of factors that could lead to low performance of teachers in the topic studied and the authors believe that some of the factors could include the type of training at teacher training institutions which involve the nature of SMK taught and the PCK acquired during training. Though the findings of this study could not be generalized to other topics taught at the level of schooling considered, it was likely that similar findings could be obtained using the same sample because the nature of the teaching strategies that were explained by teachers were not likely to be different. The observations from this study led the researchers to also conclude that the search for teachers' PCK is not yet over.

RECOMMENDATIONS

It is recommended that further studies be conducted to:

i) Explore mathematics teachers' pedagogical content knowledge in all mathematics topics of the syllabus considered
ii) Analyze the pedagogical content knowledge of mathematics teachers in equations of two variables using other theoretical frameworks.

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