 J Economics, 2(1): 1-7 (2011)

# Constrained Optimisation Technique and Management Accounting

**Oziegbe Aigbokhaevbolo**

*Department of Accounting, P.M.B. 14, Ambrose Alli University, Ekpoma Edo State, Nigeria*
*E-mail: aomaigbo@yahoo.com*

**KEYWORDS** Constraint Problems. Langrangian Multiplier. Feasible Solution

**ABSTRACT** Management accounting is an aspect of accounting that aids management to record, plan and control activities, and to help in decision - making . Most decision- making in management accounting assume that organisations have perfect freedom of choice. In reality, organisations operate under constraints such as their contractual obligation. It is against this background that this paper examines some issues involved in constrained optimisation techniques and consequently, concept, rationale and the procedures of constrained optimisation technique in making optimal decision. At the end, this paper recommends among others the need for management accounting to integrate constrained optimisation techniques into the existing accounting system.

## 1. INTRODUCTION

Management accounting is an aspect of accounting that is concerned with the provision of information to people within the organisation to help them make better decisions and improve the efficiency and effectiveness of the existing operation (Drury 2004). A good summary of usage of management accounting according to Lawrence (1996) includes (1) formulating policy, (2) planning and controlling activities, (3) decision-taking, (4) optimising the use of resources, (5) safeguarding assets, (6) cost analysis and ascertainment, (7) provision of information that will supplement the information provided by financial accountants in fin-ancial statement. From the above it can be inferred that at the centre of the management accounting task is decision-making.

Decision-making is what every decision - maker (individual, household, business organisation or government organisation) does. In organisations, decision- making is a key managerial responsibility to come to a decision. A decision can be said to the "best" course of action chosen by a decision-maker as the most effective means at his disposal for achieving goals and the "best'' solution to problems .

The fundamental issue in decision-making is making rational decisions. Making a rational decision can be defined as choosing the alternative with the best expected outcome or expected value (sometimes called mathematical expectation) using both qualitative and quantitative approaches .

According to Lagunoff and Schreft (1999), rational investors or decision makers are assumed to have rational expectations. That is, they maximise an objective function subject to perceived constraints, and they use all the information available to them, including the true probability distributions of the economy's random variables, when forming expectations.

Unfortunately, organisational decisions provided by management accounting could be described as suffering from "irrational pessimism'' and in determining the output levels it assumed that organisations have perfect freedom of choice. In reality organisations would operate under constraints such as their contractual obligation, capital rationing etc.

The implication, of course, is that irrational investors do not have rational expectations. In particular, they are assumed to optimise, but to form expectations given their subjective, and incorrect beliefs about the distributions governing random variables.

However, one main technique to obtain the best (optimal) solution to real life problems is the optimisation. Thus, the main purpose of this paper is to conceptualise and describe the procedures which are involved in making optimal decision using optimisation technique.

This paper is divided into six sections. Section one is this introductory section, section two deals on optimisation concept and rationale, section three looks at techniques for constrained optimization. While section four discuses the techniques for solving constrained optimization, section five makes recommendations and section six concludes the paper.

## 2. OPTIMISATION: CONCEPT AND RATIONALE

In the multi-disciplinary nature of operation management, the term optimization refers to the study of problems in which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within an allowed set. In simple words, it is a technique of finding the value of the independent variable(s) that minimizes or maximizes the value of the dependent variable.

As noted above an optimization problem is the problem of finding the best solution from all feasible solutions. More formally, an optimization problem A is a quadruple (I,f,m,g), where

- I is a set of instances;
- given an instance $x \in I$, f($x$) is the set of feasible solutions;
- given an instance x and a feasible solution y of x, m(x,y) denotes the measure of y, which is usually a positive real.
- g is the goal function, and is either min or max.

The goal is then to find some instance x an optimal solution, that is, a feasible solution y with

$m(x,y) = g\{m(x,y') \mid y' \in f(x)\}$.

For each optimization problem, there is a corresponding decision problem that asks whether there is a feasible solution for some particular measure $m_0$ (see http://en.wikipedia.org/wiki/ptimizationproblem 2008).

The application of optimization technique, for example assist decision makers to find the value of output that would maximizes their total revenue; some decision makers facing a constant price may want to find the level of output that would minimize the average cost and more importantly most firms may be interested in finding the level of output that would maximize their profit (Dwivedi 2002). Optimization technique uses prescriptive (dogmatic) models. According to Howard (1975), when using a prescriptive model, the goals must be specified first (maximize profit, maximize output, minimize cost or maximize lateness for example) then the best alternative is found.

Another important feature of an optimization technique is that it is a useful tool for decision makers in a no resource constraints situation as well as in a resource constraints situation. Specifically, in a no resource constraints situation, decision makers are assumed to possess unlimited recourses

For example, in the case of output maximization, firms face no resource constraint, that is they possess unlimited resources and can acquire all the inputs, finance, capital equipment, men and raw materials that are necessary to maximize output. Same is the case with cost minimization technique. The firm has all the resources to carry out production activity until average cost is minimized or cost for a given output is minimized.

Here, for example, in the case of output maximization, firms faced resource constraints, that is they possess limited resources and they cannot acquire all the inputs, finance, capital equipment, men and raw materials that they need to maximize output. Same is the case with cost minimization technique. The firms do not have all the resources to carry out production activity until average cost is minimized or cost for a given output is minimized.

In the real business world, however, the managers face serious resource constraints. For example, they need to maximize output with a given quantity of capital and labour time. As mentioned earlier one popular technique that is used to optimize the business objectives(s) under constraints is called constrained optimization techniques.

According to Dwivedi (2002), there are three very common techniques for constrained optimisation, namely, linear programming, constrained optimization by substitution and Lagrange multiplier but in practice, solving an optimisation requires hybrid of the three techniques.

## 3. TECHNIQUES FOR CONSTRAINED OPTIMIZATION

As stated earlier, the constraints are the limitations on a firm's to meet its desired objectives. Constraints may be grouped into those which have been determined within the company (endogenous) and those determined outside (exogenous). Exogenous constraints may be seen as controllable variables, for example, if the labour supply is flexible but it would obviously take time to increase the size of the plant. The exogenous variables will be to a large degree uncontrollable but may be influenced for example by (realistic) price changes and promotional effort.

In case of techniques for constrained optimization, constrained problems could be transformed into unconstrained problems with the help of Lagrange multipliers. Lagrange multipliers named after Joseph Louis Lagrange, is a method for finding the extrema of a function of several variables subject to one or more constraints; it is the basic tool in non-linear constrained optimization. Simply put, the technique is able to determine where on a particular set of points (such as a circle, sphere, or plane) a particular function is the smallest (or largest).

Formally, Lagrange multipliers compute the stationary points of the constrained function. By Fermat's theorem, extrema occur either at these points, or on the boundary, or at points where the function is not differentiable for example, starting from constrained optimization problems and depending on the dimensional space which in most cases two, the problems unconstrained problem can be solved with twice- differentiable functions, dy/dx = $f^1(x)$; $d^2y/dx^2 = f^{(n)}(x)$, Here f2 (x) is the first derivative, f2 2 (x) is the second derivative.

When the function is plotted in a graph, the first derivative at any point is the gradient, (slope of a straight line, ratio of the vertical and horizontal distances between two points on the line). For the second derivative function, the Hessian determinant matrix could be used to determine or test the existence of the relative extrema in a function with two or more variables under free or unconstrained optimization. If the Hessian is negative definite at x, then f attains a local maximum at x. If the Hessian has both positive and negative Eigen values , ë then x is a saddle point for f (this is true even if x is degenerate). Otherwise the test is inconclusive. Note that for positive semidefinite and negative semidefinite, Hessians test is inconclusive (see http://en.wikipedia.org/wiki/ptimizationproblem 2008).

## 4. APPLICATION OF TECHNIQUES FOR SOLVING CONSTRAINED OPTIMIZATION PROBLEM

We will illustrate output maximization, cost minimization and profit maximization problems.

### 4.1 Output Maximization

Output is the record of the number of goods and services produced in a particular period of time. Different terms are used to describe out put. However, output can be described as manufacturing production or as turnover. Output minimization comes up when a decision maker wishes to obtain utmost possible output subject to cost constraint. Mathematically, if $x_1$ $x_2$ represent the two quantities of inputs needed to produce a product , given existing technology, then a production function (Q) with two factors and one product can be expressed as:

$Q = f(x_1,x_2)$, .......................................... (1)

Subject to cost and cost function given as

$C = P_1 x_1 + P_2 x_2$ , .............................. (2)

The basic approach in solving the production optimisation problem is by the Lagrangian multiplier method. The problem has to be converted to form a Lagrangian function by combining the objective function and the constraint function and then solving it by the partial derivative method.

In order to formulate the Langrangian function, first set the constraint equation (1) equals to zero i.e

$C - P_1 x_1 + P_2 x_2 = 0$ .................................... (3)

second, multiply the resulting equation(3) by $\lambda$ (lambda) i.e

$\lambda [C - P_1 x_1 + P_2 x_2] = 0$ , ............................. (4)

and finally, add the resulting equation to the objective function . Thus, the Lagrangian function is formed as

$Q = f(x_1,x_2) + \lambda [C - P_1 x_1 + P_2 x_2]$ ............ (5)

Equation 5 is the unconstrained Langrangian function with three unknowns, $x_1$,$x_2$ and $\lambda$ equal to zero. The values of $x_1$,$x_2$ and $\lambda$ maximizes the output (Q) . The $\lambda$ is the Lagrangian multiplier which gives a measure of a small change in the constraint on the objective function. Since in this case we are to maximize Q we shall obtain the partial derivatives of Q with respect to $x_1$,$x_2$ and $\lambda$ and set each of them equal zero then we will obtained the coordinates at the stationary point. This will give the first order condition of output maximization in the form of three simultaneous equations as shown below:

$\frac{\partial Q}{\partial x_1} = f_1 - \lambda x_1 = 0$ .................................... (6)

$\frac{\partial Q}{\partial x_2} = f2 - \lambda x_1 = 0$ .................................... (7)

$\frac{\partial Q}{\partial \lambda} = C - P_1 x_1 + P_2 x_2 = 0$ ........................... (8)

by solving the system of simultaneous equation above by transforming and dividing Eq 6 by Eq 7 we have

$$\frac{f_1}{f_2} = \frac{P_1}{P_2} = \frac{MP\ x_1}{MP\ x_2} \quad \text{.......................................} (9)$$

or

$$\lambda = \frac{f_1}{f_2} = \frac{P_1}{P_2} = \frac{MP_{x1}}{MP_{x2}} \quad \text{.............................} (10)$$

Finally, the second condition to determine the nature of stationary point requires that the relevant bordered Hessian determinant be positive, that is,

$$\begin{pmatrix} f_{11} & f_{12} & -P_1 \\ f_{21} & f_{22} & -P_1 \\ f_{11} & f_{12} & 0 \end{pmatrix} > 0 \quad \text{…………………..} (11)$$

### 4.2 Cost Minimisation

Cost minimisation can be defined as an amount of expenditure (actual or notional) incurred or attributable to specified goods or services. Cost can also be defined as the amount of resources used for some purpose which is, therefore, the amount of liability incurred for a commodity or service ((Lucy 1993; Aigbokhaevbolo 2003). Thus, cost of production can be defined as an amount of inputs (actual or notional) incurred, attributable or used for production of goods and services. Inputs are labour, capital, raw materials, technology, purchased spare parts and other miscellaneous goods and services that are consumed in the production process. In a more practical sense, these factor inputs are reduced to the weighted average of labour and capital (Okojie 1995; Jhingan 1997; Obadan and Odusola 2002).

Labour means different things to different people. However, according to The New Webster's Dictionary (1995), to the economists and accountants labour means work as a factor of production. That is any mental or physical effort that is directed towards production of goods and services. According to Fess and Niswonger (1981), to the economists the return to labour is wages, to accountants the return to labour can be wages or salary depending on the nature of the labour. The term salary is usually applied to payment for managerial, administrative or similar services. The rate of salary is ordinarily expressed in term of a month or a year. The remuneration for manual labour, both skilled and unskilled, is commonly referred to as wages and is stated on time or piecework basis.

Capital is an important resource for production and productivity. Capital is a basket of assets or property owned by the organizations that has economic value. Capital assets can be categorized into two: fixed or physical capital assets and financial capital assets; fixed or physical capital assets consist of assets kept in the business whose benefits are beyond one year. Fixed or physical assets are usually obtained with financial capital assets. Buildings, machinery, vehicles etc. are examples of fixed or physical capital assets whereas financial capital assets are resources or property owned by the organization whose benefits are less than one year. They include cash, debtors, payment in advance (Prokopenko and North 1996; Philip 1998).

Cost minimization arises when a decision maker may wish to curtail his cost of production subject to output constraint. It means a manufacturing firm may wish to find the combination of labour cost and capital cost that minimizes cost subject to the constraint output.

Formally, if $P_1$ denotes the price of labour costs($x_1$) and $P_2$ denotes the price of capital costs($x_2$), then the total cost of production function (c) is given as:

$$C = P_1 x_1 + P_2 x_2 \quad \text{......................................} (12)$$

subject to output. Output function is given as

$$Q = f(x_1, x_2), \quad \text{...........................................} (13)$$

The basic approach in solving the cost optimisation problem is by the Lagrangian multiplier method. The problem has to be converted to form a Lagrangian function by combining the objective function and the constraint function and then solving it by the partial derivative method.

In order to formulate the Langrangian function, first set the constraint equation (1) equals to zero i.e

$$f(x_1, x_2), - Q = 0 \quad \text{......................................} (14)$$

second, multiply the resulting equation by $\lambda$,that is

$$\lambda\ f(x_1, x_2), - Q = 0 \quad \text{..................................} (15)$$

and finally, add the resulting equation to the objective function . Thus, the Lagrangian function is formed as

$$C = P_1 x_1 + P_2 x_2 + \lambda[Q - f(x_1, x_2)] \quad \text{...............} (16)$$

Equation 16 is the unconstrained Langrangian function with three unknowns, $x_1$,$x_2$ and $\lambda$ equal to zero. The values of $x_1$,$x_2$ and $\lambda$ minimizes the cost (C). The $\lambda$ is the Lagrangian multiplier. It gives a measure of a small change in the constraint on the objective function. Since in this case we are to minimise c we shall obtain the partial derivatives of c with respect to $x_1$,$x_2$ and $\lambda$ and set each of them equal zero . This will give the first order condition of cost minimization in the form of three simultaneous equations as shown below:

$$\frac{\partial c}{\partial x_1} = P_1 - \lambda f_1 = 0 \ldots\ldots (17)$$

$$\frac{\partial c}{\partial x_2} = P2 - \lambda f_2 = 0 \ldots\ldots (18)$$

$$\frac{\partial c}{\partial ë} = Q - \lambda f(x_1,x_2) = 0 \ldots\ldots (19)$$

Solving the system of simultaneous equations above by transforming and dividing Eq 18 by Eq 19 we have

$$\frac{f_1}{f_2} = \frac{P_1}{P_2} = \frac{MP\ x_1}{MP\ x_2} \ldots\ldots (.20)$$

or

$$ë = \frac{f_1}{f_2} = \frac{P_1}{P_2} = \frac{MP_{x1}}{MP_{x2}} \ldots\ldots (21)$$

Thus, equation 20 or 21 states that the value of marginal productivities must be equal to the ratio of input prices.

Finally, the second condition to determine the nature of stationary point requires that the relevant bordered Hessian determinant be negative, that is

$$\begin{pmatrix} -\lambda f_{11} & -\lambda f_{12} & -P_1 \\ -\lambda_{21} & -\lambda f_{22} & -P_1 \\ -f_1 & -f_{12} & 0 \end{pmatrix} < 0 \ldots\ldots (22)$$

## 4.3 Profit Maximisation

Profit is generally the positive deference between total revenue and total cost. According to Malomoa (1999), Drucker suggested that profit serves three purposes: (i) it measures the net effectiveness and soundness of a business's effort, (ii) it is the premium that covers the cost of staying in business- replacement, obsolescence, market and technical, risk and uncertainty and (iii) it ensures the availability of future capital for innovation and expansion, either directly by providing the means of self- financing out of retained profits, or indirectly through providing an inducement for attracting the investment of new outside capital.

Over the years, various classes of profit have emerged as the main objective of business organisations shifts from profit maximization to profit satisfaction. Under the profit maximization assumption, the owner of the firm takes all the profit, with government sector demanding for taxes for public benefits. The shareholders no longer take all the profits; this gave rise to other concepts of profit. For instance, the positive deference between total revenue and direct operating cost (cost of goods sold) is called gross profit and the gross profit less indirect expenses is called operating profit. When the operating profit is adjusted for taxes it becomes profit before tax and when the firm's tax payable is established and deducted, it becomes profit after tax (Net Profit). Finally when dividend is deducted from profit after tax, we will have retained profit.

### *4.3.1 Expressing by Equations*

Total Sales Revenue –Total Operating Cost (Cost of goods sold) = Gross Profit ......... (23)

Gross Profit –Indirect Cost = Operating Profit ................................................................ (24)

Adjusted Operating Profit – Taxes = Net Profit. ................................................................ (25)

Net Profit Divided = Retained Profit ........ (26)

The above expressions of profit are what economists called accounting profit because the perception does not take into account the implicit costs (opportunity cost). Thus the economic, profit can be expressed:

Total Sales Revenue – (Explicit Costs + Implicit Costs) ……........................................... (27)

Where explicit costs are cash outlay such as total operating cost (cost of goods sold), indirect cost and taxes. The implicit cost is the noncash outlay, the forgone income (opportunity cost) when an entrepreneur uses his factors of production. Therefore, the forgone income includes interest, salary and rent.

For the constrained profit maximization, it means a manufacturing firm may wish to maximize profit with a constraint on output.

Understanding that profit (P) is given as revenue PQ minus cost, we have a two- factor production and one product production of the form:

$Q = f(x_1,x_2)$ ........................................... (28)

and cost function given is as

$C = P_1 x_1 + P_2 x_2$ ...................................... (29)

then the ð is in a form of

$\Pi = p\ f(x_1,x_2) - P_1 x_1 + P_2 x_2$ .................... (30)

since output $= f(x_1,x_2)$ if the constraint then the Langrangian function. First set the constraint equation (1) equals to zero, that is

$f(x_1,x_2),- Q = 0$ ...................................... (31)

second, multiply the resulting equation by $\lambda$, that is

$\lambda\ f(x_1,x_2),- Q = 0$ .................................. (32)

and finally, add the resulting equation to the objective function. Thus the Lagrangian function is formed of

$ð = P\ f(x_1,x_2) - P_1 x_1 + P_2\ x_2 + \lambda[Q-f(x_1,x_2)]$. .................................................................. (33)

Equation 33 is the unconstrained Langrangian function with three unknowns, $x_1,x_2$ and $\lambda$ equal to zero. The values of $x_1,x_2$ and $\lambda$ minimizes the cost (C). The $\lambda$ is the Lagrangian multiplier. It gives a measure of a small change in the constraint on the objective function. Since in this case we are to minimise c we shall obtain the partial derivatives of c with respect to $x_1$, $x_2$ and $\lambda$ and set each of them equal zero. This will give the first order condition of cost minimization in the form of three simultaneous equations as shown below:

$$\frac{\partial ð}{\partial x_1} = Pf_1 - \lambda P_1 = 0 \quad (34)$$

$$\frac{\partial ð}{\partial x_2} = Pf_2 - \lambda P_2 = 0 \quad (35)$$

$$\frac{\partial ð}{\partial \lambda} = Q - \lambda f(x_1,x_2) = 0 \quad (36)$$

Solving the system of simultaneous equations above by transforming and dividing equation 34 by equation 35 we have

$$\frac{f_1}{f_2} = \frac{P_1}{P_2} = \frac{MP\ x_1}{MP\ x_2} \quad (37)$$

or

$$\lambda = \frac{f_1}{f_2} = \frac{P_1}{P_2} = \frac{MP_{x1}}{MP_{x2}} \quad (38)$$

Thus, equation 37or 38 states that the value of marginal product must equal the per unit input cost. Also, equation 37or 38 states the ratio of the marginal productivities of the two inputs must equal the per unit price ratio.

Finally, the second condition to determine the nature of stationary point require that the relevant bordered Hessian determinant alternate in sign as that is,

$$\frac{\partial^2 - ð}{\partial x^2_1} = Pf_{11} < 0 \quad (39)$$

$$\frac{\partial ð}{\partial x^2_2} = Pf_{22} < 0 \quad (40)$$

then

$$\begin{pmatrix} \frac{\partial^2 ð}{\partial x^2_1} & \frac{\partial^2 ð}{\partial x_1\ \partial x_2} \\ \frac{\partial^2 ð}{\partial x_1} & \frac{\partial^2 ð}{\partial x_2\ \partial x^2_1} \end{pmatrix} = p^2 \quad (41)$$

and

$$\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} > 0 \quad (42)$$

## 5. CONCLUSION

In this paper, an attempt was made to examine some issues involved in constrained optimization. Accordingly, concept, rationale and the procedures of constrained optimization in making optimal decision using optimization technique were discussed. This paper also looked at some techniques for constrained optimization as well the application of the techniques for solving constrained optimization problems to obtained feasible solutions. At the end of this paper, it can be concluded that due to internal consistency of the optimization techniques there is need for management accountant to incorporate optimization techniques to data gathering and analysis in order to improve on the quality and adequacy of information provided to organisations for decision making.

## 6. RECOMMENDATIONS

Factors of production are limited in supply therefore constrained optimization techniques in organizations is inevitable. As a result this paper recommends the following :

(1) Management must acquired the knowledge and technical capacity necessary to use constrained optimization techniques
(2) Where the constrained take the form of equalities such as equals and greater than a particular variable calculus cannot be used.
(3) Where the constrained take the form of equalities such as equals to specific number of variable calculus can be used.
(4) Where there is one equality constrained substitution can be used.
(5) Where there is more than one constrained, or if the constrained is of complex form Lagrange multiplier technique can be used.

## REFERENCES

Agbadudu AB 2004. *Calculus and Matrix for Accounting, Business and Economic Students*. Benin City: A B. Mudiaga Ltd University Press.

Aigbokhaevolo OM 2009 . *Cost Accounting.* Benin City: Ejodamen Publishers.

Dwivedi DN 2000. *Managerial Economics.* 6th Edition. New Delhi: Vikas Publishing House Pvt. Ltd.

Drury C 2004 *Management Accounting*. 6th Edition. Britain: Book Power.

Ekanem OT, Iyoha MA 1999 *Microeconomics Theory.* Benin City: Mureh Publishers.

Fess EP, Niswonger CR 1981 *Accounting Principles*. Chicago: South- Western Publishing Co.

laguunoff RO, Schreft SL 1999**.** *Financial Fragility with Rational and Irrational Exuberance.* Federal Reserve Bank of Kansas City

Gough J, Hill S 1979. *Fundamentals of Managerial Economics.* London : Macmillian

Henderson JM, Quandt I.R 1980. *Microeconomic Theory: A Mathematical Approach.* Singapore: McGraw Hill Books.

Howard K1975. *Qualitative Analysis For Planning Decisions.* London: Macdonald and Evans Ltd.

Lucy T 1993. *First Course in Cost and Management Account.* London: LP Publication.

Lawrenece S 1996. *International Accounting.* Boston: Thomas Business Press.

Malomo M 1999.*Management Accounting with Application of Quantitative Techniques*. Lagos: Chinedun Publishers Ltd.

Obadan MI, Odusola AF 2000. Productivity and Unemployment in Nigeria. *Proceedings of the Ninth Annual Conference of The ZonaL Research Units.* Abuja: Research Unit CBN, pp.186-222.

Official Home of Wikipedia http://en.wikipedia.org/wiki/ptimizationproblem (Retrieved 28 November 2008).

Okojie CEE 1995**.** Human apital investment for productivity**.** *The Nigerian Economics and Financial Review,* 1(1): 43- 68.

Prokopenko J, North K 1996. *Productivity and Quality Management: A Modular Programme*. Geneva: International Labour Office.

The New Webster's Dictionary 1995.*The New Webster's Dictionary of the English Language.* New York: Lexicon Publications Inc.