PRINT: ISSN 0975-1122 ONLINE: 2456-6322
Int J Edu Sci, 13(2): 231-246 (2016)
DOI: 10.31901/24566322.2016/13.02.10

Exploration of the Levels of Mathematical Proficiency Displayed by Grade 12 Learners in Responses to Matric Examinations

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KEYWORDS Computational Fluency. Connections. Facility of Mathematics. Logic. Non-routine. Routine. Script Analysis

ABSTRACT This paper presents grade 12 learners' levels of mathematical proficiency exhibited in response to matric mathematics examination questions in the Gauteng Province in South Africa. Document analysis was used on n = 90 learners' scripts on four topics, namely, sequences and series, differential calculus, analytical geometry and trigonometry. Findings revealed that on the four topics, learners were most proficient in examination questions on analytical geometry that tested only procedural knowledge. However, a majority of learners were not proficient in examination questions that tested conceptual knowledge on analytical geometry. Furthermore, most learners were not proficient in examination questions on sequences and series, differential calculus as well as trigonometry on both procedural and conceptual knowledge. Such findings implied that it was most likely that learners left high school without being proficient on these topics, which incidentally formed the bulk of the prerequisite mathematics course in engineering and other natural science programs at university.

INTRODUCTION

Grade 12 Mathematics achievements in the Republic of South Africa have been a concern irrespective of recent educational reforms (DBE 2013). Low achievements in mathematics, physical science and accounting have been noted during the release of Matric results by the Department of Basic Education (DBE) in recent years (Chen 2014; DBE 2013; Mji and Makgato 2006). This paper seeks to identify levels of mathematical proficiencies exhibited in grade 12 learners' responses to examination questions.

According to Chisholm (1999), the final examination's focus is to test the knowledge of the syllabus, is used as a filter for university admission, assesses a learner's achievement of learning outcomes, and also holds departmental officials and educators accountable for teaching and learning. However, Sieborger and Macintosh (1998) aver that it is impossible to structure final examinations such that it is totally valid or reliable due to political issues developed by parents, employers, educational authorities and tertiary institutions.

A study by Stein et al. (1996) investigated well-structured mathematical tasks and revealed that they promote thinking and reasoning, include multiple solution strategies and multiple representations of mathematics. Their study also revealed that well-structured mathematical tasks include memorization and use of algorithms with or without concept understanding, and promote complex thinking and reasoning (p.460). They contend further that well-structured mathematical tasks promote explanation, justification and conjecturing. Schoenfeld (2007) argued that well-structured mathematical tasks must address the following postulates, that is, they should ensure students use of multiple solution strategies in their response to examination questions, ensure students produce mathematical explanations and justifications, and ensure students engage in high level thinking and reasoning. These characteristics are embedded intrinsically in the construct of mathematical proficiency, which students might exhibit in their responses (Cragg and Gilimore 2014; Greenes 2014; Guberman and Leikin 2013).

The philosophical assumptions that guided this study came from Schoenfeld's (2007) notion of assessing mathematical proficiency, that is, 'what you test is what you get (WYTIWYG)' (p.72). Mathematical proficiency that learners exhibit in response to the matric examinations is those tested by the examinations.

Background

The 2003 Trend in International Mathematics and Science Study (TIMSS) study showed that South Africa was the lowest in Mathematics achievement amongst the participating countries, even those that were poorly resourced as compared to South Africa (Leung 2014, 2005; Reedy 2006). In their study, Kiwanuka et al. (2015) identified various factors that that were predictors of mathematics achievement such as, good classroom assessment, parental support, socioeconomic status, previous mathematics achievement, gender and peer influence. However, such mathematics achievement was not quantifiable in terms of proficiency levels. A study by Taylor and Vinjevold (1999) that conducted observations in classrooms identified challenges in mathematics education in South Africa such as, domination of teacher centered methods, lack of teaching and learning structures that promoted higher order reasoning, prevalent of misuse of everyday examples by teachers in their teaching and low teachers' morale and self-esteem. This study did not consider observable proficiency levels on the effect of mathematics learning. However, such challenges might influence the strands of mathematical proficiency that learners exhibit, which was the focus of this study. Maharaj et al. (2015) noted that mental conflicts are directly linked to the strands of mathematical proficiency.

Maharaj et al. (2015) revealed that students' proficiency was enhanced by using website material and that non-mathematical terminology used in class contributed negatively to students' achievement of proficiency in mathematics. Other findings from a study (Luneta 2015a) pointed at challenges that lead to lack of mathematical proficiency as being, the limited time that teachers give learners to respond to mathematical tasks, tasks set by teachers not meeting the required standard that engage learners in higher order and critical thinking, and mathematical meaning being lost when teachers in rural schools use vernacular to explain mathematical concepts. Another study by Mbonambi and Bensilal (2014) indicated that most learners did not have basic algebra knowledge required to handle problems and reach the required solution. Furthermore, the researchers advised that attempts to enhance algebraic proficiency should be enforced at lower grades. Ally (2011) reported that the promotion of mathematical proficiency in South African schools as weak and unstructured.

Theoretical Framework

In this study the notion of mathematical proficiency was used as a lens to view grade 12 learners performance in their response to examination questions. This study used four strands of mathematical proficiencies, that is, procedural fluency, conceptual understanding, strategic competence, and adaptive reasoning, and according to Kilpatrick et al. (2001), the strands are interconnected, intertwined and interrelated. These four strands of mathematical proficiency were found to be relevant to document analysis that the study used in analyzing the learners' responses.

Procedural fluency and conceptual are conceptualized as forms of mathematical knowledge whilst strategic competence and adaptive reasoning were seen as mathematical skills. In the context of this study these strands were in various ways identified in learners' responses to the final examination questions that were under consideration (Kilpatrick et al. 2001).

Bartell et al. (2013) described mathematical proficiency as the advancement in mathematical skills and knowledge. Kilpatrick et al. (2001) defined mathematical proficiency as knowledge, expertise, competence and the facility of mathematics. Groves (2012) viewed mathematical proficiency as the ability to engage in mathematics with understanding, computing, applying, reasoning and engaging.

Procedural fluency describes the learner's ability of carrying out mathematics procedures in a manner that is flexible, accurate and efficient (Suh 2007; Kilpatrick et al. 2001). Procedural fluency also involves knowing when to use mathematical procedures, 'knowing how' of mathematical knowledge, which is a learner's ability to quickly recall and accurately execute procedures (Graven and Stott 2012; Zakaria and Zaini 2009). Procedural fluency is often called computing, which involves engaging in practice with procedures and manipulation of numbers, knowing 'how to do it' of mathematical knowledge as well as algorithm and computational fluency (Bass 2003; Russell 2000).

The second strand of mathematical proficiency, conceptual understanding refers to learners' ability to comprehend mathematical procedures, concepts relations and operations (Groves 2012; Kilpatrick et al. 2001; Rittle-Johnson and Alibali 1999). Zakaria and Zaini (2009) argued that conceptual knowledge is mathematics that is rich in relationships, interconnections between ideas, which explain and give meaning to mathematical procedures. An observation here is that conceptual understanding focuses on the ability to work with a variety of procedures simultaneously and with procedural fluency, learners work with procedures in isolation. In their elaboration of conceptual knowledge, Schneider and Stern (2010) explained that conceptual understanding is knowing the 'why' of mathematics knowledge when connecting related algorithms and knowledge items. McCormick (1997) argued that when learners exhibit the ability to link algorithms and knowledge items, conceptual knowledge is seen as general and abstract mathematical knowledge of the key principles and their interrelations in the mathematics domain. Schoenfeld (2007) described conceptual understanding as the explicit or implicit understanding of the principles that govern the mathematics domain coupled with the interrelations amongst parts of knowledge in the domain.

The third strand of mathematical proficiency, strategic competence, is learner's skills of formulating, representing and solving mathematical problems (Kilpatrick et al. 2001). Strategic competence is sometimes called problem solving, which is the skill of using computations to solve routine and non-routine mathematical problems that require learners to be fluent in procedural knowledge and some degree of conceptual knowledge (Groves 2012; Naroth and Luneta 2015). According to Schoenfeld (2007), strategic competence has two components of skills, the generative and evaluative skills, which are also components of problem solving, which requires some degree of creativity and flexibility.

The last strand of mathematical proficiency used in this study is adaptive reasoning, which refers to the learner's skills for logical thought, reflection, explanation and justification (Kilpatrick et al. 2001). It also involves the learner's capacity to think logically about relationships between mathematics concepts and relations (Aaron and Herbst, 2015; Ally and Christiansen, 2013; Amir-Mofidi, 2012; Kilpatrick et al. 2007; Suh 2007).

Objectives

This paper explores the levels of mathematical proficiency that learners exhibit in response to four matric examination questions namely, sequences and series, differential calculus, analytical geometry and trigonometry. The researchers wanted to know the extent to which these learners exhibit levels of mathematical proficiency that were tested by the selected matric examination questions.

METHODOLOGY

This study gathered qualitative data, which was analyzed thematically as well as using descriptive statistics. Qualitative research generates theory through interpreting evidence and data gathered from direct quotations and open-ended questions (Volkwein 2001). The data used in this study provide a thick description of a learner's responses to final examination questions. Document analysis in form of question by question was used to view levels of mathematical proficiency that learners exhibited in their responses to examination questions (Creswell 2014).

The scripts were collected by the University of Johannesburg Faculty of Education project called Script Analysis. This study used ninety scripts from a batch of one thousand scripts, forty-five from the 2007 National Senior Certificate examination paper one and forty-five from paper two. The scripts were sampled using convenience sampling and only those with substantial responses to the chosen questions were used. The scripts were in batches of fifty and the researchers simply selected scripts in this range of marks, group A (GA) from 0-50, group B (GB) from 51-90 and group C (GC) from 90-200 marks. The researchers used these categories only for the purpose of managing the learners' responses.

The data was analyzed using the instruments developed from theory on strands of mathematical proficiency with reference to learners' performance in the examination questions. This means that for both paper one and paper two, thirty scripts were from learners who got from zero and fifty, group A (GA), thirty scripts were from learners who got from fifty one to ninety, group B (GB), and thirty scripts were from learners who got from ninety to two hundred, group

C (GC). This helped the researchers identify mathematical proficiencies that various ranges of achievement displayed in their response to the selected examination questions.

Cohen and Manion (1994) stipulated that the standard rules of content analysis included, the size of the data to be analyzed, that is, is it a line, a paragraph or a phrase? What were the units of meaning? Are themes inclusive of all categories, or mutually exclusive? Data defined precisely with the mention of its properties, and lastly, was that data fitted in some category? Categories must be exhaustive. In the context of this study the type of the data were the learners' responses to examination questions that showed the spectrum of proficiencies that learners displayed and the size was the number of responses to the questions in the study. The categories were predetermined based on the meaning of mathematical proficiencies that were found in the review of literature. This provided the deductive analysis of the data because the categories were predetermined. The indicators for mathematical proficiencies were designed to address as much proficiencies as possible that learners displayed in their response to examination questions and these are shown in Table 1.

RESULTS

Forty-five learner responses on the chosen questions were analyzed from the 2007 higher grade mathematics finalexamination and the strands of mathematical proficiencies were unveiled using codes as shown in Table 1. The researchers ascertain that it was challenging to analyze the strands in isolation, and therefore, the researchers subscribed to Kilpatrick et al.'s (2001) view that they are intertwined and interconnected. A question-by-question analysis was used to analyze learner's responses. In the analysis, the researcher categorized learners' proficiencies into strands of three levels—not proficient, moderate proficient and proficient. Learners' responses and a few questions below outline how the codes were generated from the learners' responses to the chosen examination questions.

Examination Questions on Sequences and Series

Below is an example of a question on sequences and series that learners responded to during the examination:

"Question 5.1.2: Calculate the sum of all natural numbers less than or equal to 200, excluding those that are multiples of 7."

Recommended Answer in Marking Guideline

$S_{28} = \frac{n}{2}[2a + (n-1)\,d]$ (formula, learner to select from formula sheet)

$= \frac{28}{2}$

Table 1: generic codes for mathematical proficiency

Mathematical proficiencies	*Codes*	*Performance by learner*
Conceptual Understanding	CU0	No response to the question, learners left blank spaces.
	CU1	Learners' show no knowledge of connecting procedures.
	CU2	Learners perform major errors when connecting procedures.
	CU3	Learners perform minor errors when connecting procedures.
	CU4	Learners connects and reconnects procedures fluently.
Procedural Fluency	PF0	No response to the question, learners left blank spaces.
	PF1	Learners' computations are incorrect.
	PF2	Learners perform major errors in computations.
	PF3	Learners perform minor errors in computations.
	PF4	Learners carry out computations fluently.
Strategic competence	SC0	No response to the question, learners left blank spaces.
	SC1	No skills of problem solving.
	SC2	Learners exhibit limited skills of problem solving.
	SC3	Learners exhibit acceptable skills of problem solving with no creativity.
	SC4	Learners are proficient in problem solving with creativity.
Adaptive Reasoning	AR0	No response to the question, learners left blank spaces.
	AR1	No skills of mathematical reasoning.
	AR2	Learners use representations to display logical thought and justification.
	AR3	Learners use only logic with minor errors to justify responses.
	AR4	Learners used only logic to justify responses.

= [2(7) + (28-1)7] (substitution)

= 2842 (simplification, correct answer, sum of multiples of 7)

S_{200}=

$\frac{200}{2}$ [2(1) + (200-1)1] (substitution)

= 100[2 + 199]

= 20100 (simplification, correct answer, sum of all multiples equal or less than 200)

S_n = 20100 – 2842 = 17258 (difference, removing multiples of 7).

Examples of Learners' Responses

The example below shows four examples of learner's work on this examination question on sequences and series.

Learner A

5.1.2 $Sn = \frac{n}{2}(a + l)$

$\frac{200}{2}(a + 7)$

$= 100(a + 7)$

$= 100a + 700$

$-700 = 100a$

$a = 7$

Learner B

5.1.2 $Sn = \frac{n}{2}[2a + (n - 1)d$

$S_{200} = \frac{n}{2}[2(1) + (n - 1)7]$

$400 = n[2 + 7n - 7]$

$= 7n^2 - 5n - 400$

$= (7n - 50)(n + 8)$

$n = \frac{50}{1}$ or $n = -8$

Learner C

5.1.2 $200 - 28 = 172$

$Sn = \frac{172}{2}[2(1) + (172 - 1)(1)]$

$= 86[2 + 171]$

$Sn = 14878$

$S_{28} = \frac{28}{2}[7 + (28 - 1)7]$

$S_{28} = 2744$

$= 14878 - 2744$

$= 12134$

Learner D

5.1.2 $S_n = \frac{n}{2} = [2a + (n - 1)d]$

$S_{28} = \frac{28}{2}[2(7) + 27(7)]$

Sum of multiples of 7 is 2842

$S_{200} = \frac{200}{2}[2(1) + (199)1]$

=

$100(2 + 199)$

20100

$S_{200} - S_{28}$

$= 20100 - 2842$

$= 17258$

The sum of all numbers excluding the multiples of 7 is 15258.

A summary of the learners' responses to examination questions on sequences and series is summarized in terms of codes that outline learners' responses as shown in Table 2.

The codes in the learner responses appeared in strands, and there were three levels of mathematical proficiency. The first level of not proficient consisted of the following codes PF0-SC0 and PF1-SC1 (procedural knowledge) and PF0-SC0-CU0 and PF1-SC1-CU1 (conceptual knowledge). There were 70.8 percent learners' responses for all the questions on sequences and series that were not proficient. The second level moderate proficient, had the following codes, PF2-SC1, PF2-SC2, PF3-SC2, PF3-SC3 (procedural knowledge) and PF2-SC1-CU1, PF3-SC2-CU1 and PF3-SC2-CU2 (conceptual knowledge). There were 14.3 percent learners that were moderate proficient for all questions on sequences and series. The last level or proficient had the following codes, PF4-SC4 (for procedural knowledge) and PF4-SC4-CU4 (for conceptual knowledge). There were 14.9 percent of learners that were proficient in all questions on sequences and series. From these findings it is safe to say that questions 5.1.1, 5.1.2, 5.2.1, 5.4.1 and 5.4.2 tested procedural knowledge, whilst question 5.2.2 and 5.3 tested conceptual knowledge.

Explanation of Codes

Learner A's response is one of the ten who were not proficient on this question and were coded PF1-SC1. The code means PF1 that the learner chose the wrong formula from the formula sheet and the resultant solution was wrong. The researchers classified this learner as one who had no knowledge of the procedure. For the code SC1, substitution and simplification were considered wrong because of the use of an incorrect formula.

Learner B was one of twelve that were coded PF2-SC2. The code PF2 means that Learner B chose the correct formula however incorrectly used the procedure of finding the sums. For the code SC2, other values substituted correct however '200' is substituted incorrectly and since the follow-up procedure is wrong, the resultant substitution is wrong.

Table 2: Summary of analysis of matric learners' responses questions on sequences and series

Matric 2008 questions on sequences and series	*Codes for levels of mathematical proficiency*			
Examination questions	*Not Proficient*	*Moderate Proficient*	*Proficient*	*Total*
5.1.1 How many of the first 200 natural numbers are multiples of 7.	(12)PF0-SC0 (13)PF1-SC1	(1)PF2-SC1 (1)PF3-SC3	(18)PF4-SC4	45
5.1.2 Calculate the sum of all natural numbers less than or equal to 200, excluding those that are multiples of 7.	(17)PF0-SC0 (10)PF1-SC1	12)PF2-SC2 ((2)PF3-SC2	(4)PF4-SC4	45
5.2.1A candidate guesses that 'r' is 2. Explain without calculations why this guess is correct	(9)PF0-SC0 (28)PF1-SC1	(4)PF2-SC2	(4)PF4-SC4	45
5.2.2 Calculate the value of 'r'.	(12)PF0-SC0-CU0 (19)PF1-SC1-CU1	(2)PF2-SC1-CU1 (7)PF3-SC2-CU2	(5)PF4-CU4-SC4	45
5.3 A ball falls from a height of 10m, bounces up 6m and continues to bounce ¾ of the height of the previous bounce. Calculate after how many bounces the height reached by the ball will be less than 1cm.	(19)PF0-SC0-CU0 (20)PF1-SC1-CU1	(2)PF3-SC2-CU1	(4)PF4-SC4-CU4	45
5.4.1 Determine the general (n^{th}) term of the series.	(10)PF0-SC0 (19)PF1-SC1	(8)PF2-SC2	(8)PF4-SC4	45
5.4.2 Write the series in sigma notation.	(11)PF0-SC0 (24)PF1-SC1	(6)PF3-SC2	(4)PF4-SC4	45
Totals	223	45	47	315
Percent	70.8	14.3	14.9	100

Learner C was one of two that were coded PF3-SC2. The code PF3 means that the learner first found the correct difference by 'subtracting 28 from 200', chose correct formula, and substituted correctly, however simplification was inconsistent with the procedure of finding the sum. SC2 means that the substitution in the formula was correct, followed by wrong simplification.

Learner D is one example of four who were coded PF4-SC4. The code PF4 means that the learner chose a correct formula and used the procedure appropriately to find the sum. The code SC4 means that learner D substituted correctly in the formula and simplified correctly for both sums (S_{28} and S_{200}), and then calculated the difference between sums accurately to reach the correct answer.

Examination Questions on Differential Calculus

Question 6.2.2 (paper 1) on the introduction of differential calculus is below

6.2 *Determine* $\frac{d}{dx}$ *in each of the following:*

6.2.2 $y\,(x + 1) = 3x^2 + 4x + 1$

Suggested Answer in Marking Guideline

6.2.2 $y\,(x + 1) = 3x^2 + 4x + 1$

$y\,(x + 1) = (3x + 1)(x + 1)$ (finding factors)

$y = 3x + 1$ (simplification)

$\frac{dy}{dx} = 3$ (finding the derivative)

Examples of Learners Responses

Below are examples of learners' responses for question 6.2.2 on differential calculus.

Learner A

6.2.2 $y(x + 1) = 3x^2 + 4x + 1$

$y(x + 1) - 3x^2 - 4x - 1 = 0$

$= yx + y - 3x^2 - 4x - 1$

$-3x^2 + 4x - yx - y + 1 = 0$

$\therefore 3x^2 + 4x - yx - y + 1 = 0$

Learner B

$y(x + 1) = 3x^2 + 4x + 1$

$y = \frac{(3x + 1)(x + 1)}{(x + 1)}$

$y = 3x + 1$

$\frac{dy}{dx} = 3$

In Table 3 is a summary of the learners' responses to examination questions the introduction of differential calculus is summarized in terms of codes that outline the learners' responses as shown in Table 2.

The first level of mathematical proficiency in the examination questions on differential calculus is not proficient, and the following codes were visible in learners' responses, PF0-SC0, PF1-SC1 (for procedural knowledge) and PF0-SC0-CU0, PF1-SC1-CU1 (for conceptual knowledge). There were 61.9 percent learners that were not proficient. For the level moderate proficient, the following codes were identified, PF2-SC2, PF3-SC2 (for procedural knowledge), and PF2-SC1-CU1, PF2-SC2-CU1, PF2-SC2-CU2 and PF3-SC3-CU3 (for conceptual knowledge). There were 12.6 percent learners who were moderate proficient. For the level proficient, the following codes were identified, that is, PF4-SC4 (for procedural knowledge) and PF4-SC4-CU4 (for conceptual knowledge). There were 25.5 percent learners that were proficient. Questions 6.1.1, 6.3.2, 6.4.1, 6.4.2, 7.1.2, 7.2.1 and 7.2.2 tested procedural knowledge, and question 6.2.1, 6.2.2, 6.3.1, 7.1.1 and 7.2.3 tested conceptual knowledge.

Explanation of Codes

Above are two examples of learner's responses for examination questions on differential calculus that demanded conceptual knowledge. Learner A's response was one of the seventeen classified as "not proficient" and was one of those coded PF1-SC1-CU1. The code PF1 means, that the Procedure is incorrect. For the code SC1, simplification is wrong and the learner did not find the derivative. The code CU1 means that the learner did not connect procedures, factorization and finding derivative.

Learner B's response was one of the eleven classified as proficient, and thus coded PF4-SC4-CU4. The code PF4 means that the procedure of finding the derivative was correct. For the code SC4, factorization and calculating the derivative is correct. For CU4, the learner comprehended

Table 3: Summary of analysis of matric learners' responses on introduction to differential calculus

Matric 2008 questions on sequences and series	*Codes for levels of mathematical proficiency*			
Examination questions	*Not Proficient*	*Moderate Proficient*	*Proficient*	*Total*
6.1.1 Given $f(x) = x^5$, determine the derivative from first principles.	(6)PF0-SC0-CU0 (3)PF1-SC1-CU1	(3)PF2-SC1-CU1 (11)PF3-SC3-CU2	(22)PF-SC4-CU4	45
6.1.2 Hence write down $\frac{dy}{dx}[5f(x)]$.	(11)PF0-SC0 (15)PF1-SC1		(19)PF4-SC4	45
6.2.1 $y = \sqrt{x}(\sqrt{x^3 - 2x})$	(7)PF0-SC0-CU0 (16)PF1-SC1-CU1	(1)PF2-SC1-CU1 (2)PF2-SC2-CU2	(19)PF4-SC4-CU4	45
6.2.2 Determine $\frac{dy}{dx}$ for $y(x+1) = 3x^2 + 4x + 1$	(7)PF0-SC0-CU0 (17)PF1-SC1-CU1	(2)PF2-SC2-CU1 (8)PF2-SC2-CU2	(11)PF4-SC4-CU4	45
6.3.1 Given, $f(x) = x^{\frac{2}{3}}$, determine the value of 'a' for which $f'(a) = 1$.	(11)PF0-SC0-CU0 (16)PF1-SC1-CU1	(7)PF2-SC2-CU2	(11)PF4-SC4-CU4	45
6.3.2 For which value(s) of x, is $f'(x)$ not defined?	(14)PF0-SC0 (19)PF0-SC0		(12)PF4-SC4	45
6.4.1 At what rate is the production cost changing when the 800th ounce of gold is produced.	(20)PF0-SC0 (9)PF1-SC1	(14)PF3-SC2	(2)PF4-SC4	45
6.4.2 The production cost is increasing at that moment. How do you know this?	(18)PF0-SC0 (22)PF1-SC1		(5)PF4-SC4	45
7.1.1 Calculate the co-ordinates of A, B, C.	(5)PF0-SC0-CU0 (8)PF1-SC1-CU1	(14)PF2-SC2-SC1	(18)PF4-SC4-CU4	45
7.1.2 Hence use the graph to determine the values of x which satisfy the following inequalities; $f'(x) < 0$ and $f(x) < 0$.	(27)PF0-SC0 (6)PF1-SC1	(6)PF2-SC2	(6)PF4-SC4	45
7.2.1 Express 'h' in terms of x.	(18)PF0-SC0 (16)PF1-SC1		(11)PF4-SC4	45
7.2.2 Show that the cost, C, of the material is given by $C(x) = 10x^2 + 540x^{-1}$	(26)PF0-SC0 (12)PF1-SC1	(4)PF2-SC2	(3)PF4-SC4	45
7.2.3 Calculate the value of x for which the cost of the material is minimum.	(25)PF0-SC0-CU0 (8)PF1-SC1-CU1	(2)PF2-SC2-CU2	(10)PF4-SC4-CU4	45
Totals	**362**	**74**	**149**	**585**
Percent	**61.9**	**12.6**	**25.5**	**100**

procedures correctly, and these are factorization and finding derivative.

Examination Questions on Analytical Geometry

For question 1.1.3 (paper 2) learners responded to the following question.

The points, A (-1; 3), B (4, -2) and C (-3; k) are the Cartesian plane

AB = BC, CM is the median with M on ABM. D is the y-intercept of straight line AB.

1.1.3 Hence if k = -3, determine:

(b) Whether F, the y-intercept of the median CM, is the point of concurrency of the medians of triangle ABC

Suggested Answer in Marking Guideline

As x = 0 then y = (the point is on the y-axis and substituting with zero)

F (0,) (co-ordinates of F)

Mid-point of AC 'N'

N = [(substitution in the formula for mid-point)

N = (-2; 0) (co-ordinates of mid-point)

M_{BN} = (calculating gradient of line BN)

= (answer, the gradient of BN)

y = (substituting in the slope-intercept form for straight line)

y = (resultant equation)

As x = 0 then y = (x-intercept and substituting with zero)

Then F is the point concurrent (it is where the mid points meet).

Examples of Learners' Responses

Below are examples of learners' responses for question 1.1.3b on analytical geometry.

Learner A

1.1.3(b) $x = 0\ y = 7$

$\frac{y_1 + y_2}{2} = y$

$= \frac{-3 + \frac{1}{2}}{2}$

$= \frac{-2\frac{1}{2}}{2}$

$-1\frac{1}{4} = y$

$\left(0, 1\frac{1}{4}\right)$

Learner B

(b) $F\left(0; -\frac{2}{3}\right)$

$CF = \sqrt{(-3-0)^2 + \left(-3 + \frac{2}{3}\right)^2}$

$= \sqrt{9 + \frac{16}{9}}$

$CF = \sqrt{\frac{27}{9}}$

$FM = \sqrt{(\frac{3}{2} - 0)^2 + (\frac{1}{2} + \frac{2}{3})^2}$

$= \sqrt{\frac{9}{4} + \frac{25}{36}}$

$= \sqrt{\frac{106}{36}}$

$AF = \sqrt{(-1-0)^2 + \left(3 + \frac{2}{3}\right)^2}$

$= \sqrt{\frac{14}{3}}$

$BF = \sqrt{(4-0)^2 + \left(\frac{-2}{3} + 2\right)^2}$

$= \sqrt{16 + \frac{16}{9}}$

$BF = \sqrt{\frac{160}{9}}$

$\therefore$ F the y – inter of median CM is not the point of concurrency of ΔABC

Learner C

(b) F lies at point (0: y)

$9y = 7xx - 6$ sub (0; y)

$9y = 7(0) - 6$

$y = \frac{-6}{9}$

$\therefore y = -2$

$F(0; \frac{-2}{3}$

$CF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_{2)}^2}$

$= \sqrt{[0-(-3)]^2 + \left[\frac{-2}{3} - (-3)\right]^2}$

$= \frac{\sqrt{130}}{3}$

$$FM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\frac{3}{2} - 0\right)^2 + [\frac{1}{2} - (\frac{-2}{3})]^2}$$

$$= \frac{\sqrt{130}}{6}$$

$$\frac{FM}{CF} = \frac{\sqrt{130}}{6} \times \frac{3}{\sqrt{130}}$$

$$= \frac{1}{2}$$

$\therefore$ ratio FM : CF = 1: 2

$\therefore$ F is a centroid of ΔABC

The first level of mathematical proficiency for examination questions on analytical geometry, not proficient, had the following codes, PF1-SC1 (for procedural knowledge), and PF0-SC0-CU0, PF1-SC1-CU1 and PF1-SC1-CU1-AR1 (for conceptual knowledge). There were 24.4 percent learners that were not proficient in examination questions on analytical geometry. For the level moderate proficient, the following codes were identified, PF2-SC2-CU2, PF3-SC2-CU2 and PF3-SC3-CU3-AR3 (for conceptual knowledge). There were 18.9 percent learners that were moderate proficient in examination questions on analytical geometry. For the last level, proficient, the following codes were identified, PF4-SC4 (for procedural knowledge) and PF4-SC4-CU4-AR4 (for conceptual knowledge). There were 56.7 percent of learners that were proficient in examination questions of analytical geometry. Question 1.1.1a tested procedural knowledge, and questions 1.1.1b, 1.1.2, 1.1.3a, 1.1.3b and 1.2 tested conceptual knowledge.

Explanation of Codes

Learner A's response was one of five, which were classified as "not proficient" and coded PF1-SC1-CU1-AR1. The code PF1 means that the procedure was not known by this learner. For the code SC1, the learner could not follow any problem solving procedures to solve the problem. The code CU1 means the learner could not connect procedure, which were gradient, x-intercept and mid-point to prove concurrency. For the code AR1, the learner had no skills of proving for concurrency.

Learner B was one of eleven responses for learners that were classified moderate proficient and coded PF2-SC2-CU2-AR2. The code PF2 means the learner chose the correct procedure, distance formula, to prove concurrency, however the use of the procedure had errors that did not lead to the required conclusion. For the code SC2, the learner substituted correct in the distance formula, however the simplification was wrong and did not lead to the required conclusion. The code CU2 means the learner comprehended two distances to prove concurrency, which eventually did not lead to the required

Table 4: Summary of analysis of matric learners' responses on introduction to analytic geometry

Matric 2008 questions on analytical geometry	*Codes for levels of mathematical proficiency*			
Examination questions	*Not Proficient*	*Moderate Proficient*	*Proficient*	*Total*
1.1.1a Determine the co-ordinates of the point M.	(2)PF1-SC1		(43)PF4-SC4	45
1.1.1b Determine the co-ordinates of the point D	(1)PF0-SC0+CU0 (4)PF1-SC1-CU1	(5)PF3-SC2-CU2	(35)PF4-SC4-CU4	45
1.1.2 Show that k = -1 or k = -3	(7)PF0-SC0-CU0 (20)PF1-SC1-CU1	(3)PF3-SC2-CU2	(15)PF4-SC4-CU4	45
1.1.3a Hence, if k = -3 determine the equation of the median CM	(1)PF0-SC0-CU0 (3)PF1-SC1-CU1	(3)PF2-SC2-CU2	(38)PF4-SC4-CU4	45
1.1.3b Whether F, the y-intercept of the median CM, is the point of concurrency of the medians of the triangle ABC.	(12)PF0-SC0-CU0-AR0 (5)PF1-SC1-CU1-AR1	(11)PF2-SC2-CU2-AR2 (6)PF3-SC3-CU3-AR3	(11)PF4-SC4-CU4-AR4	45
1.2 Determine the numerical value of p if the straight line defined by 2y = px+1.	(4)PF0-SC0-CU0 (7)PF1-SC1CU1	(23)PF3-SC2-CU2	(11)PF4-SC4-CU4	45
Totals	66	51	153	270
Percent	24.4	18.9	56.7	100

conclusion. The code AR2 means the learner used two distances to prove median, an acceptable proof, however were made errors that did not warrant an acceptable proof.

Learner C's response was one of the eleven classified as proficient and coded PF4-SC4-CU4. For the code PF4, the learner used correct procedure, distance formulae to correctly prove concurrency. The code SC4 means that the learner substituted correct in the formulae and the simplification led to the correct conclusion of proving concurrency. For the code CU4, the learner connected procedures of two distances correct to prove concurrency.

Examination Questions on trigonometry

For question 6.1.1 (paper 2) learners responded to one of the question below.

In the diagram alongside, BD is a chord of circle ABCD. BD = BC = p

6.1.1 Show that CD = -2p.cos A

Suggested Answer in the Marking Guideline

BD = BC = p (given in the question)

< C = <D_2 (base angles of an isosceles)

<C + <A = 180^o (angles in a cyclic quad sum)

<C = 180^o - <A (making angle C the subject of the formula)

$BD^2 = BC^2 + DC^2 – 2(BC).(DC).\text{Cos}(180^o – A)$ (cosine rule)

$p^2 = p^2 + DC^2 - 2p(DC)(-\text{Cos}A)$ (substitution)

$0 = DC^2 + 2p(DC)\text{Cos}A$ (simplification)

$-DC^2 = -2p(DC)\text{Cos}A$ (simplification)

DC = -2p Cos A (answer, proved)

Examples of Learners' Responses

Below are examples of learners' responses for question 6.1.1 on trigonometry.

Learner A

6.1.1 $CosA = \frac{DB}{BC} + OC$

$CosA = \frac{P}{P} + DC$

$\frac{PCos\ A = P + PDC}{P}$

$PDC = PCosA - P$

$PDC = P(CosA - 1)$

$D = -2P.CosA$

Learner B

6.1 $a^2 = b^2 + c^2 - 2AD.AB.CosB$

$P^2 = AB^2 + AD^2 - 2AD.ABCosA$

$CD^2 = P^2 + P^2 - 2.2pCosB$

$CD = 2(AB^2 + AD^2 - 2AD.ABCosA) - 4pCosB$

Learner C

6.1.1.) $BD = BC = p$

$\therefore < C = < D$

$< C + < A = 180^o$

$BD^2 = BC^2 + DC^2 - 2(BC)(DC)Cos(180^o - A)$

$p^2 = p^2 + DC^2 - 2p(DC)(-CosA)$

$O = DC^2 + 2p(DC)CosA$

$-DC^2 = 2p(DC)CosA$

$DC = -2pCosA$

Explanation of Codes

The examples above show three learners' responses to a question on trigonometry that demanded conceptual knowledge. Learner A's response was one of those thirteen classified as "not proficient" and coded PF1-SC1 CU1. For the code PF1, the learner chose the wrong procedure resulting in the solution being wrong. The code SC1 means all substitutions and simplifications considered wrong due to wrong procedure used. For the code CU1, connection of procedures considered wrong due to the use of wrong procedure.

Learner B's response was one of the twelve classified as "moderate proficient" and was coded PF2-SC2-CU1. The code PF2 means the learner chose the correct procedure however execution of the procedure the procedure was wrong. For the code SC2, the learner substitutes correct but simplification is wrong. The code CU1 means the learner B did not show that procedures needed to be connected to solve the problem, and these were the cosine rule and the concept of angles of a cyclic quadrilateral.

Learner C's response was one of the two classified as proficient and coded PF4-SC4-CU4. The code PF4 means the learner used correct procedure to get to the required answer. The code SC4 means that learner C substituted and simplified correct, which lead to the correct answer. The code CU4 means the learner comprehended procedures correct and these are cosine and angles of a cyclic quadrilateral.

The first level of mathematical proficiency for examination questions on trigonometry, not proficient, had the following codes, PF0-SC0, PF1-SC1 (for procedural knowledge), and PF0-SC0-CU0, PF1-SC1-CU1, PF0-SC0-CU0-AR0 and PF1-SC1-CU1-AR1 (for conceptual knowledge).

Table 5: Summary of analysis of matric learners' responses on trigonometry

Matric 2008 questions on trigonometry		*Codes for levels of mathematical proficiency*		
Examination questions	*Not Proficient*	*Moderate Proficient*	*Proficient*	*Total*
3.1 Calculate the numerical value of $Cos^2(180^0-\theta)$ +Cot θ	(3)PF0-SC0-CU0 (10)PF1-SC1-CU1	(8)PF3-SC2-CU1 (14)PF3-SC3-CU3	(10)PF4-SC4-CU4	45
3.2.1 Prove that tan α +tan β =1	(6)PF0-SC0-CU0- AR0 (26)PF1-SC1-CU1-AR1	(5)PF2-SC2-CU1-AR1(8)	PF4-SC4-CU4- AR4	45
3.2.2 Hence calculate the size of β if it is given that tan α = /3.	(15)PF1-SC1-CU1	(5)PF2-SC2-CU1	(25)PF4-SC4-CU4	45
3.3 Simplify: sin $(\theta + 45^0)$. Sec 585^0. Cosec $(\theta - 90^0)$	(3)PF0-SC0-CU0 (10)PF1-SC1-CU1	(22)PF2-SC2-CU1	(10)PF4-SC4-CU4	45
4.1 Solve for x if sin 3x = Cos $(x + 180^0)$ and x ∈ $[-90^0; 90^0]$.	(3)PF0-SC0-CU0 (10)PF1-SC1-CU1	(22)PF2-SC2-CU1 (5)PF3-SC3-CU2	(5)PF4-SC4-CU4	45
4.2 By using axes provided sketch graphs of the curves of *f* and g for x $[-90^0 ; 90^0]$, clearly show the co-ordinates of all turning points, end points and intercepts with axes.	(3)PF0-SC0-CU0	(17)PF2-SC2-CU2	(25)PF4-SC4-CU4	45
4.3.1 Both f(x) and g(x) are increasing as x increasing.	(13)PF0-SC0 (12)PF0-SC0	(5)PF3-SC3	(15)PF4-SC4	45
4.3.2 Sin 3x < -Cos x	(15)PF0-SC0 (14)PF1-SC1	(12)PF2-SC2	(4)PF4-SC4	45
5.1 determine, rounded off to one decimal digit the general solution of 3Cos x + 10 Sin x + 5 = 0	(6)PF0-SC0-CU0 (20)PF1-SC1-CU1	(8)PF3-SC2-CU2	(11)PF4-SC4-CU4	45
5.2.1 Prove the identity, Cot 2θ + cosec 2θ = Cot θ.	(16)PF0-SC0-CU0-AR0 (5)PF1-SC1-CU1-AR1	(13)PF2-SC2-CU1 -AR1	(11)PF4-SC4-CU4-AR4	45
5.2.2 For which values of θ $[-180^0; 0^0]$ is the identity undefined.	(22)PF0-SC0-CU0 (8)PF1-SC1-SC1	(10)PF2-SC2-SC2	(5)PF4-SC4-CU4	45
6.1.1 Show that CD = -2p.CosA.	(18)PF0-SC0-CU0 (13)PF1-SC1-CU1	(12)PF2-SC2-CU1	(2)PF4-SC4-CU4	45
6.1.2 Explain why the angle A $[90^0;180^0]$.	(22)PF0-SC0-CU0 (23)PF0-SC0-CU0			45
6.2 Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove that: $q^2 = p^2 + r^2 - 2 (p)(r)Cos\ \theta$.	(21)PF0-SC0-CU0-AR0 (4)PF1-SC1-CU1-AR	(9)PF2-SC2-CU2 -AR2	(11)PF4-SC4-CU4 -AR4	45
6.3.1a Write the length of AO.	(9)PF0-SC0(2)PF1-SC1		(34)PF4-SC4	45
6.3.1b Write the length of BT.	(7)PF0-SC0(3)PF1-SC1		(35)PF4-SC4	45

Table 5: Contd...

Matric 2008 questions on trigonometry	*Codes for levels of mathematical proficiency*			
Examination questions	*Not Proficient*	*Moderate Proficient*	*Proficient*	*Total*
6.3.2 Hence determine the length of PB in terms of the trig ratio and BT.	(8)PF0-SC0(8)PF1-SC1	(21)PF3-SC2	(8)PF4-SC4	45
6.3.3 If it is further given that angle APB = 71^0, calculate the distance from A to B to the nearest metre.	(10)PF0-SC0+CU0 (10)PF1-SC1-CU1	(5)PF2-SC2-CU1	(20)PF4-SC4-SC4	45
Total	37	193	239	810
Percent	846.7	23.8	29.5	100

There were 46.7 percent learners that were not proficient. The second level, moderate proficient, learners exhibited the following codes, PF2-SC2, PF3-SC2, PF3-SC3 (for procedural knowledge) and PF2-SC2-CU1, PF2-SC2-CU2, PF3-SC3-CU2, PF3-SC3-CU3, PF2-SC2-CU1-AR1, PF2-SC2-CU2-AR2 (for conceptual knowledge). There were 23.8 percent learners that were moderate proficient. For the level, proficient, learners exhibited the following codes, PF4-SC4 (for procedural knowledge) and PF4-SC4-CU4, PF4-SC4-CU4-AR4 (for conceptual knowledge). There were 29.5 percent learners that were proficient in examination questions on trigonometry.

DISCUSSION

The analysis of the learners' scripts revealed that the learners' proficiencies ranged from "not proficient" to "moderate proficient" and "proficient" as shown in Figure 1.

The researchers viewed the strands of mathematical proficiencies as discursive learners' responses that were located between three placeholders shown in Figure 1. The strands of mathematical proficiency shown in Tables 2, 3, 4 and 5 showed the journey learners embark on their quest to become proficient in mathematics. In this journey, learners' levels of proficiency may be viewed using the continuum as shown in Figure 1. One other observation was the existence of the (SC0-4) levels that were intertwined with either procedural knowledge or conceptual knowledge. This confirms findings from a study by Naroth and Luneta (2015) that problem solving is central to mathematics learning that is either procedural or conceptual.

There were three categories of mathematical proficiencies that were exhibited by learners in their response to examination questions on all four topics analyzed for questions that demanded either procedural (coded PF-SC, 1-4) or conceptual knowledge (coded PF-SC-CU 1-4). For learners who were on the level not proficient, the strands showed learners who left blank spaces and those that used wrong mathematical procedures resulting in their responses being mathematically wrong.

For the level moderate proficient, there were a variety of indicators that showed this category, as some learners only wrote the correct formula, substituted correct and simplified wrong, some chose correct formula, substituted correct, simplified correct and wrote wrong conclusion, some chose correct formula, failed to comprehend procedures, some comprehended correct and failed to justify their answers. On the continuum, these learners' level of proficiency was either ranging between moderate proficient and not proficient or moderate proficient and proficient.

For learners who were proficient, they used correct procedures, comprehended procedures correctly, substituted correct, simplification was correct and reached the correct answers or conclusions.

Responses for Sequences and Series Examination Questions

A bulk of learners' responses fell in the category of not proficient, meaning that they either wrote incorrect responses or left blank spaces, which contradicts findings by Ally (2011) that in grade 6 mathematics classrooms the strand procedural fluency was prevalent. Further, learners' incorrect responses and blank spaces were regarded by Luneta (2015b) as a deficit of basic concepts in the topic. The existence of codes that flashed moderate proficiency indicated that for questions that examined procedural knowledge there were less numbers falling within the range not proficient and moderate proficient (1PF2-SC1) (See Table 2). Most learners who were moderate proficient fell within the range moderate proficient to proficient. This pattern was also applicable to the questions that examined conceptual knowledge. This is an indication that these learners struggled with examination questions that either examined automatized knowledge that occurred with minimal conscious attention or maximum conscious attention. Schneider and Stern (2012) as well as McCormick (1997) referred to such learners as those that did not have the capacity to transform their conscious attention to higher mental functions.

Responses for Differential Calculus Examination Questions

Previous studies have indicated that learners and even teachers are keen to apply instructions that lead to procedural knowledge as opposed to those that lead to conceptual knowledge (Maharaj et al. 2015; Zakaria and Zaini 2009; Simpson and Zakaria 2004). A study by Froneman et al. (2015) justified the need to balance students' procedural and conceptual knowledge. The results for learners' responses on differential calculus show that majority of the students were not proficient in questions that examined both procedural and conceptual knowledge. Although there were signs of learners being proficient in differential calculus examination questions, there were still those that grappled with the knowledge of key concepts and were between the levels not proficient to moderate proficient and moderate proficient to proficient. The conceptual nature of differential calculus examination questions that Luneta and Makonye (2010) identified seemed to be the problem for learners' mastery of basic differential calculus concepts such as, differentiating from first principle and finding the derivative. In these cases, learners struggled with simplifying exponents, finding the limit and use of correct notation. Some of the questions had fractions and a study by Dhlamini and Kibirige (2014) as well as Gabriel et al. (2013) indicated that learners use their alternative conceptions, which are wrong to solve problems on fractions. This was attributed to the conceptual nature of fractions, which was lacking in learners' knowledge. A study by Gordillo and Godino (2014) pointed such learning deficiency to integrating numeric and algebraic concepts as well as integrating students' conceptions to those of algebra as a whole.

Responses for Analytical Geometry Examination Questions

Luneta (2015b) reported that most learners in his study operated at level 2 of the van Hiele's hierarchy of geometrical thought when the demand in the examination questions required learners to navigate between levels 1 to 4. It is noted here that while Luneta (2015b) viewed

|--► |--► |

Not Proficient Moderate Proficient Proficient

Fig. 1. Mathematical proficiencies continuum found in learners' scripts

learners' responses to transformational geometry, and the findings contradict findings of this study in that more than half of the learners sampled in this study were proficient in analytical geometry for questions that examined both procedural and conceptual knowledge, the fact is still that a good number of learners could not understand the concepts, hence lacked conceptual knowledge. There was also still a fraction of learners that were not proficient and those that ranked between moderate proficient and proficient. There were no learners who were stranded in the continuum between not proficient and moderate proficient. Foster (2013) who also used data from transformational geometry pointed out that learners' difficulties in answering geometry question may be attributed to the frequent use of unfamiliar contexts.

Responses for Trigonometry Examination Questions

Arnold (2006) pointed out that trigonometry is characterized by rich connections of related concepts and algorithms. When students' are exposed to the learning of trigonometry through experiential learning enhanced their proficiency, as they were able to connect their learning to their experiences (Tuna 2013). However, Weber et al. (2008) emphasize that trigonometry learning is dominated by algorithmic fluency at the expense of rich mathematics that is characterized by intensive understanding. The current study revealed that almost half the learners were not proficient in basic concepts needed to learn trigonometry. Some learners were moderate proficient, which shows that although they could perform basic concepts needed for trigonometry, they struggled with the conceptual nature of trigonometry. Those that were proficient mastered basic concepts as well as proficiency in the conceptual structure of trigonometry.

CONCLUSION

From this study it is safe to say that it was most likely that a majority of learners that left high school were not proficient in matric examination sequences and series, differential calculus and trigonometry that demanded both procedural and conceptual knowledge. Pointers may be directed to teaching procedurally, as literature revealed that teachers themselves struggled to teach conceptually and lacked basic conceptual understanding and related procedures. The study also revealed that there were learners who left high school with moderate proficient and grappled with the conceptual nature of the topics as well as some complex procedures. This explains why most of learners leaving high school struggle in tertiary courses that demanded the knowledge of sequences and series, such as engineering.

Furthermore, the study revealed that it is more likely that more learners exhibit proficiency on examination questions on analytical geometry that demanded procedural applications than conceptual understanding. It was also observed that a substantial number of learners who left high school were moderate proficient in examination questions on analytical geometry that demanded both procedural and conceptual knowledge. The overall observation made from the study was that learners were procedurally proficient in analytical geometry than the other chosen examination questions such as trigonometry but conceptual knowledge was lacking throughout the four topics.

RECOMMENDATIONS

The researchers would like to recommend that teaching needs to focus on equipping learners with basic skills as well as related concepts and algorithms that will enable them to be proficient in matric examination questions. These proficiencies need to be developed in the lower grades and nourished throughout the schooling years. It is further recommended that future studies on script analysis should include interviews where learners elucidate further on the errors and misconceptions that are hard to conclude from the scripts.

LIMITATIONS

The study had a small number of scripts that were analyzed and this made it difficult to generalize from the findings. However, the results may be used to indicate significant levels of mathematical proficiency among learners. Some of the learners' responses needed more information from the learners and it was impossible to interview the learners to find reasons to most of their responses. Therefore, most of the findings could not be used as pointers to some deficit of certain levels of mathematical proficiency.

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Paper received for publication on October 2014
Paper accepted for publication on April 016