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# Teaching Methods and Learners' Concept Formation, Development and Integration in Geometry: Assessing the Relationship

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**ABSTRACT** This paper sought to determine the dominant concepts and approaches used by learners in dealing with selected geometry problems and establish the significance of using mind webs on the overall performance of learners. A descriptive research design, which utilised a mixed-methods approach, was adopted to elicit data from 74 high school learners. Permission to conduct the study was rendered by school principals and mathematics heads of the participating schools. Inferential statistics and content analysis were used to analyse data. The study established the learners' widespread poor geometric concept formation, development and integration. Non-geometric and mathematically incorrect approaches were dominantly used to arrive at answers. The findings also showed that learners who presented their thinking processes on paper averagely performed better. The paper recommends ways to improve classroom practices arguing that an omission or poor development of one learning stage will heinously affect the proper development of all the succeeding stages.

## INTRODUCTION

The lack of mathematics prerequisite skills at tertiary level has been recognised as an issue since the late 1970s and is known as the 'mathematics problem' (Rylands and Coady 2009). Participation in cross-national surveys comparing evidence of student performance in reading literacy, mathematics and science competencies (Divjak et al. 2010) and subsequently, every country's ranking in achievement in such examinations should be of significance to education stakeholders of each country. The implication of this report suggests that there is a need for the improvement in the instructional design of mathematics in the public school system (White 2007 in Sriphai et al. 2011) to facilitate the students' learning process in order to improve the results (Sriphai et al. 2011).

The issue of 'mathematics problem' is a serious one even in developed countries (Wilson and MacGillivray 2007). The desire to understand and identify factors that may have meaningful and consistent relationships with mathematics achievement has been shared among national policymakers and educators around the world (Wagemaker 2003). Vinson (2001) argues that effective mathematics teachers know that they must follow the modes of learning as presented by Bruner so that students are provided with concrete experiences that form the basis for pictorial and symbolic mathematics learning.

Psychology and educational theory had a long tradition of research in learning styles (Cassidy 2004). There is evidence that learning styles of each learner tend to be different, and hence, a single mode of instructional delivery may not provide sufficient choices, engagement, social contact, relevance, and context needed to facilitate successful learning and performance (Singh 2003). One of the most enduring effects on education had been the search for individual differences that could explain and predict variations in student achievement (Wang and Jin 2008). The hope is that pedagogical methods can be designed in a way that will capitalize on different learning styles (Sriphai et al. 2011).

The strategies that will familiarize the students with the contents of instruction, empower them with sufficient level of mathematical proficiency, increase their interest and enhance their active participation in the subject and also efficacious in improving their interaction with the environment are highly needed (Areelu and Akinsola 2014). A typical model of good practice recognised by many Japanese educators and teachers is the 'problem-solving lesson' in the mathematics classroom (Stigler and Hiebert

1999). The problem-solving lesson is structured and progressed through four distinct phases, namely, presentation of the problem, developing a solution, progress through discussion and summarizing the lesson (Burghes and Robinson 2009).

Many teaching methods that are hotly debated in the United States vary among the six higher-achieving countries. For example, the Netherlands uses calculators and real world problem scenarios quite frequently. Japan does neither. Yet both countries have high levels of student achievement (Stigler and Hiebert 2004). It can henceforth be concluded from the above scenarios that mathematics achievement depends more on certain approaches in the development of learners to become high achievers. The fact that complete usage or otherwise of calculators and real world scenarios to achieve highly in mathematics implies that there is more to mathematics teaching, conceptualisation and application than only the presence or absence of certain gadgets that aid mathematical manipulation.

## Instructional Approaches in Teaching and Learning Geometry

Geometry is the 'study of shapes, their relationships, and their properties' (Bassarear 2012:463). According to Stoker (2003:11), "learning is strongly and necessarily linked to teaching". Instructional practices to develop geometry understanding should encompass the range of learners' levels in that grade. This means that the learners require multiple and diverse activities to enable them to progress through the levels of abstraction (Battista 2007). Van Hiele (1986) states that the inability of many teachers to match instruction with their learners' levels of geometrical understanding is a contributing factor to their failure to promote meaningful understandings in this topic.

Questions of how learners learn mathematics and how they should be taught inevitably invoke speculation as to what constitutes an effective teaching method. Although no single method of teaching can be said to be 'best' for all learners and for all learning (McInerney and McInerney 2002; Stoker 2003), it is important to listen to pupils since there is a difference between what one wants children to learn and what they actually take from the lessons. Understanding the student's mind during problem solving improves the way a teacher gets closer to understanding his/her pupils (Makina and Wessels 2009).

Research evidence has indicated that discrepancies in the mathematics performance of learners internationally can be explained in terms of differences in their classroom instructional experiences (Sullivan et al. 2009). Nieuwoudt and van der Sandt (2003) found that Grade 7 teachers and prospective teachers lacked the geometry content knowledge requisite for them to be successful teachers. Thus, teachers with inadequate geometry content knowledge tend to exacerbate the problems that learners experience in the subject. "However, 'specialized content knowledge' is content knowledge that teachers require in order to do their work, which usually involves a deeper level of understanding of concepts, so that teachers can 'unpack' or understand the learners' mathematical ideas" (Brodie and Sanni 2014:89).

The content of 3D geometry encompasses the fundamentals of the representation system, which in turn entails learning about projections and manipulating models in order to be able to represent spatial objects in a 2D plane (da Costa 2014). According to González and Herbst (2006), geometry is the only high school subject in which students routinely deal with the necessary consequences of abstract properties and in which students are held accountable for reading, writing and understanding mathematical proofs.

In geometry the communication of information at different levels of reasoning of the sender (the teachers) and the receiver (the student) become a major cause of misconception (Lim 2011). When the teacher operates and communicates at different levels of geometric thought than those of the students, concepts are not understood or acquired fully. It is necessary for teachers to know their students' level of geometrical thought and to operate at those levels (Luneta 2015). Most mathematics teachers in South Africa do not have the appropriate skills, content knowledge, as well as the pedagogical content knowledge necessary to be effective in a mathematics classroom. Most mathematics teachers do not seem to have the knowledge and instructional skills required to explain concepts, but rather their teaching consists of algorithms that students are instructed to follow (Centre for Development in Education 2010; Luneta 2015).

The current paper aims to provide an insight into the kind of geometry classroom instructional practices that might have contributed to the learners' levels of geometric understanding in the selected studied South African high schools. This was prompted by the view that South African learners who have completed primary school should have reached Van Hiele's (1986) thinking level two (Feza and Webb 2005), that is, they should be "able to describe and represent the characteristics and relationships between 2D shapes and 3D objects in a variety of orientations" (Department of Education 2002: 6). This is based on the fact that as the learning process follows several stages, an omission or poor development of one stage will heinously affect the proper development of all the succeeding stages.

### Visualization Techniques Enhancing Traditional Teaching Methods

There are various teaching theories including constructivist teaching, existentialism and multiple intelligences theory. According to Moore et al. (2002), an understanding of educational theories can provide a common vocabulary for discussions about teaching, clarify the intent of instructional techniques, stimulate thinking, and enliven the daily experience of teaching. All these theories inform teaching strategies and have their merits and demerits but expanding on that subject is beyond the scope of this paper. In response to that, recent researches have established numerous visualization and non-visualization techniques that enhance learning.

There are countless visualization techniques for ideas mapping that use graphics for items, concepts, and arrows to indicate relationships such as mind mapping, cognitive mapping, conceptual diagram, visual metaphor, dialog mapping, semantic mappings and clustering (Âhlberg 2004; Eppler 2006). Although these learning strategies may differ in efficacy and applicability, they are all based on a conceptual framework called the constructivist theory of learning, which states that meaningful learning, or learning with understanding, occurs when adult learners assimilate new information within their existing frameworks (Bodner 1986 in D'Antoni et al. 2010). In the ensuing section, the researchers briefly describe the two prominent visualization techniques widely used in education, that is, mind maps and concept maps, then finally contextualize mind webs within that continuum.

### *Relevance of Mind Maps in Learning Mathematics*

Brinkmann (2003a) draws attention to the similarities between the structure of mind maps and the structure of mathematics. Both are depicted in a tree-like structure and this emphasizes the usefulness of mathematical issues as topics for mind maps. Several uses were identified for mind maps in mathematics education including organizing information, as a memory aid, to foster creativity and show connections between mathematics and non-mathematical concepts. In one study, Brinkmann (2003b) established that the tool was especially beneficial for students who were not good in mathematics as it is through creating a map that they first see connections between mathematical concepts.

Different from mind map, mind mapping means the technique for visualizing these relationships among different concepts has distinctive features over concept mapping in terms of its colors and free form. By using such pictorial and graphical design flourishes, mind mapping can make learning and teaching more vivid and thus can promote memory retention as well as enhance the motivation of the learners. Mind mapping thus promotes creative thinking, and encourages "brainstorming" (Liu et al. 2014). When it comes to free form and unconstrained structure of mind mapping, Martin (2011) believes that if so, there are no limits on the ideas and links that can be made, and there is no necessity to retain an ideal structure or format.

Mind maps and concept maps have a hierarchical structure and are produced following conventions (Novak 1990; Buzan 1993; Brinkmann 2003a). For mind mapping, these involve placing the topic in the center of the page or screen. Primary branches are drawn for each major idea linked to the topic. Keywords indicating the major ideas are written directly onto the links. From the primary branches further sub-branches for secondary ideas (subtopics) are drawn. The principle is that ideas should move from the abstract to the concrete. In mind mapping, each main branch builds up a unit with its sub-branches. For the sake of simplicity, connections between sub-branches of different main branches are not drawn (Brinkman 2003a).

Mind maps can be used as a teaching tool to promote critical thinking (McDermott and Clarke 1998). The ability to integrate information by finding valid relationships between concepts allows students who construct either mind maps or concept maps to reach a metacognitive level (Willingham 2007). Mind maps can facilitate learning in a variety of ways. Goodnough and Woods (2002) discovered that partakers in their study perceived mind mapping as a fun, interesting and motivating approach to learning. Several of these participants attributed the fun aspect to the opportunity to be creative when creating mind maps through lots of choice in color, symbols, keywords and design. Research by D'Antoni and Zipp (2005) found that, from a pool of 14 physical therapy students, 10 agreed that the mind map technique enabled them to better organize and integrate the material presented in their course.

For the reason that mind maps enable the learning and learning environments to be more effective since mind maps may enable the lobes responsible for the different activities of the brain to operate simultaneously (Seyihoglu 2013), the special technique of mind mapping, which uses both sides of the brain and has them working together, is of benefit to mathematical thinking. The left hemisphere of the brain is better suited for analytic deduction and arithmetic, and the right hemisphere for spatial tasks, such as geometry. The constant emphasis in mathematical education is on rules and algorithms, which are usually sequential, and may prevent the development of creativity and spatial ability (Pehkonen 1997). A study by Toi (2009) shows that mind mapping can help children recall words more effectively than using lists, with improvements in memory of up to thirty-two percent. Similarly, according to a study conducted by Farrand et al. (2002), mind mapping improved the long-term memory of factual information in their participants by ten percent.

### *Relevance of Concept Maps in Learning Mathematics*

Mathematics educators have long used paper and pencil tests as tools to assess learning. However, the need for a better way to represent the learners' conceptual understanding has led to the development of concept maps as an alternative assessment tool (Novak and Cañas 2006). The various developments currently underway should all help bring one closer to the time when educational practice will rely chiefly on empirical evidence rather than a combination of tradition and fads (Rohrer and Pashler 2007). Concept maps provide the student with a different means of demonstrating understanding, and the assessor with an additional opportunity to witness how the student connects ideas and groups or organizes information. In other words, concept maps effectively reveal the overall integrated knowledge of the learner (Varghese 2009). The definition for concept maps is given by Novak and Cañas (2006:1) as follows.

*Concept maps are graphical tools for organizing and representing knowledge. They include concepts, usually enclosed in or boxes of some type, and relationships between concepts indicated by a connecting line linking two concepts. Words on the line, referred to as linking words or linking phrases, specify the relationship between the two concepts. The result of linking two concepts is a proposition […] Propositions are statements about some object or event […] Propositions contain two or more concepts connected using linking words or phrases to form a meaningful statement.*

Concept maps are diagrams that represent relationships among concepts and concept mapping is a tool that visually displays the knowledge structure of given topics and the connections between these structures (Tuan and Thuan 2011). Traditional concept maps include labeled concepts, directional arrows, linking words, lines suggesting hierarchical relationships, graphic representation of concepts and propositions conveying relationship among different concepts (Wheeldon and Faubert 2009).

Âhlberg (2004) strongly supports the view that there is no need to follow some unnecessarily complex rules in Novak's standards and proposed some elements of an improved method of concept mapping. For instance, he suggested that many words can be included in a concept label instead of short verbal labels, that it is not only when links are horizontal or are read upwards that arrows are used but all links between concepts have arrowheads to show in which direction the connection from one concept to another is to be read, that multimedia resources can be inserted to concept maps, and numbers may be included to clarify the order in which the propositions should be read, and so on. Âhlberg's (2008) view is that although hierarchies are natural ways of presenting human knowledge, there is need to consider how concepts are linked to each other in one's thinking.

In terms of simplicity, spontaneity and speed in creating the resulting map, concept maps are more complex, free form with various clusters so they take longer to develop whereas mind maps are simpler, fix on a single conceptual center and faster to create. This comes down to the point that concept maps have web representation while mind maps have a radiant structure. In terms of emphasis, concept maps focus on the "clarity of display and making explicit the relationship between ideas" by employing occasional icons whereas the "artistic layout" with the maximum use of attractive, colorful pictures, shapes, images and so on, is what mind maps stress (Brightman 2003: 8).

### *Contextualizing Mind Webs as Mathematics Learning Tools*

According to Tuan and Thuan (2011), concept maps express relationships of multiple concepts both in a hierarchical manner and network representation by using arrows, whereas mind maps are restricted to radiant or tree structures with one central node in the center and many branches emanating from the center. As a result, a mind map has only one main concept and it reflects what you think about a single topic, while a concept map may have a set of concepts to present a system view. According to Âhlberg and Ahoranta (2002:119), "a concept map is an accurate representation of the main features of cognitive structure, while the mind map is an ordered association map open to multiple interpretations because a concept map presents ideas accurately not just hints as in mind map". Mind webs, therefore, are a variant to mind maps and concept maps. They may be in the form of a plan involving first, writing the steps/stages to be followed in answering a given question then coming up with the diagrammatic view of the modifiable steps during working, and finally answering the question guided by steps or diagram presented earlier. The researchers' view of mind webs is that they are an infusion of mind maps and concept maps as they are multidirectional tools that have the radial structure of mind maps in which the main concept is centrally positioned with nodes radiating from the center. However, unlike a concept map with a major topic/concept at the top, mind webs have instead several concepts surrounding the main topic/concept, which are linked in a web structure form like a concept map bordered by broad integrating learning topics. They are relevant in answering a formative or summative assessment question and involve rigor in bringing about the interconnectivity of the numerous interlinked concepts involved in arriving at the correct answer. Commonly, mind maps, concept maps and mind webs are all tools intended to simplify teaching and learning in a different manner to traditional classroom practice.

### Statement of the Problem

Mathematics teaching and learning is one of the most excruciating activities in classrooms worldwide. This emanates from the fact that the subject is partly involving and demands a significant amount of time and effort to master it. As such, there is need to deviate from traditional teaching methods and find alternative motivating ways of teaching mathematics that enhance its understanding. This paper, therefore, focused on how the use of mind webs may improve mathematics teaching while dealing with multiple-concept geometry problems.

### Purpose of the Study

The study sought to determine the impact of using mind webs and measure that against the traditional classroom teaching practices. Stigler and Hiebert (2004) contend that if one wants to improve student learning, one must find a way to improve teaching in the average classroom. Even slight improvements in the average classroom can positively affect millions of students.

The research questions to be answered were:

1. What are the dominant concepts and approaches used by learners in dealing with selected multiple-concept geometry problems?
2. To what extent can mind webs be relevant to deal with selected multiple-concept geometry problems?

### Research Objectives

The paper sought to ascertain the dominant concepts and approaches used by learners in dealing with selected multiple-concept geometry problems as well as determining the extent to which mind webs could be relevant in dealing with such geometry problems.

## Theoretical Framework

This paper is premised on principles advocating for the provision of the kind of education, which promotes what Jacobs et al. (2007) call 'relational thinking'. According to Jacobs et al. (2007), thinking relationally is different from applying a collection of tricks or memorizing a set of mathematical properties. It is a way of reasoning that is not linked to particular procedures or number combinations, and children who think relationally identify number relations and reason about which transformations make sense in a particular problem. This is also in line with Harel and Tall's (1991) 'necessity principle'. According to Harel and Tall (1991:41):

*This principle states that the subject matter has to be presented in such a way that learners can see its necessity. For if students do not see the rational for an idea (for example, a definition of an operation, or a symbolization for a concept), the idea would seem to them as being evoked arbitrarily, and it does not become a concept of the students.*

This study was an action-based research activity in which towards the end of the program, the investigated sample and the control group were examined using a single investigative assessment task that was administered to help establish the impact of the program on the group. Denzin and Lincoln (1998) assert that action research comprises research initiated by teachers or other education practitioners, conducted within the environment of the practitioner, typically small-scale, and highly contextually based. Such studies are commonly concerned with pragmatic outcomes such as improving learning, gaining a deeper understanding of classroom practice or situated learning (Keeves 1998). It was a type of action research in which the first researcher served as the teacher of record for the investigated learners. The first researcher had been dealing with this sample group for nine months as a teacher-researcher on this paper.

# RESEARCH METHODOLOGY

## Research Design

This paper adopted a descriptive research design within a mixed-methods approach to assess the significance of using mind webs in enhancing learners' understanding of concepts in answering multiple-concept geometric questions. The effect of the mind web development teaching style was assessed on the overall comparative performance of the learners in mathematics.

## Participants

Seventy-four participants took part in this study. The experimental group comprised 20 Grade 11 learners taught by the teacher-researcher at one school. Experimental participants were a mixed-ability group dominated by average to below average learners in mathematics. The control group constituted 44 Grade 11 and 10 Grade 12 learners from five other schools that participated in the final cross-comparative assessment.

## Instrument

A single question administered in the final summative assessment that solicited for quantitative and qualitative data was used to collect data. Makina and Wessels (2009) argue that summative assessments are used to make a determination about overall learning that has been achieved. Qualitative data was gathered from the part of the question in which learners were to develop the mind web or illustrate the steps they were to take to deal with each part question. Quantitative data was gathered from the actual and marked working done by learners on their scripts. The task was presented thus: *Referring to Figure 1.*

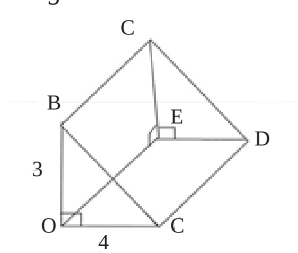


**Fig. 1. Summative assessment instrument question**
*Source:* Kariyana 2015

*Step 1: Develop a mind web or illustrate the steps you would take to deal with each part question·*

*Step 2: Show the working (applying and modifying your mind web or illustrated steps where necessary) to arrive at your answer.*

*If O is the origin and ABCD is a square:*

*(a) Find the perimeter, surface area and volume of the right-angled triangular prism OABCDE.*

*(b) Determine the coordinates of each vertex of OABCDE. Hence or otherwise, sketch OABCDE on the graph paper provided. Use a scale of 2 cm to 2 units.*

**Reliability and Validity**

To ensure reliability, Cronbach's alpha coefficient was utilised while a pilot study was employed to ensure validity of the question. The calculated difficulty index (p) was 0.61 and the discrimination index (D) was 0.27, reflecting that it was a moderately difficulty question that had desirable discrimination ability (Kubiszyn and Borich 2007). It was hence adopted as it was.

**Data Analysis**

Inferential statistics was used to analyse quantitative data while content analysis was used to analyse qualitative data.

**Ethical Considerations**

In written form, communication was made to all the Grade 11 learners, teachers and the school management team (SMT) of the launch of such a program at the teacher-researcher's school, as well as to principals, mathematics heads of department and learners of the rest of the participating schools. All rendered permission to conduct the study.

**A Factor Worth Considering**

The most significant factor to consider in this study is that this assessment task was the last and only Section C question worth 20 marks in Round 2 of the Olympiad-type assessment of a mathematics inter-school competition organized and coordinated by the first researcher. Unlike other Olympiad assessments, the thrust was that learners were to show working in this and other parts selected. The examination paper was administered by the contact persons identified in all participating schools on the same day. All participants had passed Round 1. In the 100 marks examination answered within one and a half hours, learners might have failed to find enough time to focus on the problem. It was administered without prior warning to all participants as the intention was to get the most accurate possible results. The following results emerged.

## RESULTS

The next section presents findings of the study in relation to the research questions.

**Dominant Concepts and Approaches Used by Learners in Dealing with Selected Geometry Problems**

The first question was intended to evoke the learners' understanding of measurement concepts while the second question was targeted at evoking the learners' knowledge about geometric concepts. The ability to integrate these two was critical for successfully getting to the correct answers. Table 1 summarizes the concepts and approaches mostly employed by learners in dealing with the given geometry problem.

Table 1 depicts that there were significant differences in responses between the experimental group and the control group. It emerged that most respondents from the experimental group demonstrated better reasoning skills and orderly presentation of work was observable compared to their counterparts. Experimental respondents also showed an edge over their counterparts in their application of geometric and measurement concepts to solve the problem.

Data also shows that there was higher consistency among the experimental group of mentioning the concepts such as the Pythagoras Theorem, formulae of 2D and 3D figures and the concept of gradient, compared to their counterparts. The approach by most respondents after failing to identify the concepts was to utilize the unmentioned concepts. Deviations were significantly higher for the control group regarding these two aspects. Ninety percent of experimental participants mentioned and utilized formulae for perimeters and areas of 2D figures while 66.7 percent of the control group utilized this concept, which was mentioned by fifty percent. A similar but weak trend emerged regarding surface area and volume.

Most participants in Table 1 faced difficulties to understand the coordinate system and scale, and hence could not identify coordinates

**Table 1: Dominant concepts and approaches used by learners in dealing with the given geometry problem (N=74)**

| *Process variable* | *Experimental group (N=20)* | | | | *Control group (N=54)* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Yes* | | *No* | | *Yes* | | *No* | |
| | *F* | *%* | *F* | *%* | *F* | *%* | *F* | *%* |
| Mention of Pythagoras Theorem | 17 | 85.0 | 3 | 15.0 | 12 | 22.2 | 42 | 77.8 |
| Utilisation of Pythagoras Theorem | 19 | 95.0 | 1 | 5.0 | 25 | 46.3 | 29 | 53.7 |
| Mention of correct formulae of perimeters and areas of figures | 18 | 90.0 | 2 | 10.0 | 27 | 50.0 | 27 | 50.0 |
| Utilisation of perimeter and area concepts | 18 | 90.0 | 2 | 10.0 | 36 | 66.7 | 18 | 33.3 |
| Mention of correct formulae of surface area and volume | 14 | 70.0 | 6 | 30.0 | 5 | 9.3 | 49 | 90.7 |
| Utilisation of surface area and volume concepts | 18 | 90.0 | 2 | 10.0 | 12 | 22.2 | 42 | 77.8 |
| Justified identification of coordinates of O, A and B | 12 | 60.0 | 8 | 40.0 | 12 | 22.2 | 42 | 77.8 |
| Mention and use of gradient method to find $m_{AB}$ | 8 | 40.0 | 12 | 60.0 | 7 | 13.0 | 47 | 87.0 |
| Correct identification of $m_{BC}$ | 6 | 30.0 | 14 | 70.0 | 1 | 1.9 | 53 | 98.1 |
| Correct use of gradients and properties of OABCDE to find coordinates of C, D and E | 3 | 15.0 | 17 | 85.0 | 0 | 0.0 | 54 | 100.0 |
| Establishment of (generally) a mind web | 4 | 20.0 | 16 | 80.0 | 0 | 0.0 | 54 | 100.0 |
| Correct use of trial and error method (no working) | 4 | 20.0 | 16 | 80.0 | 4 | 7.4 | 50 | 92.6 |
| Correct use of the vertices established earlier to sketch OABCDE | 3 | 15.0 | 17 | 85.0 | 1 | 1.9 | 53 | 98.1 |
| Understanding of scale | 6 | 30.0 | 14 | 70.0 | 4 | 7.4 | 50 | 92.6 |
| Correct order of naming figure after sketching | 7 | 35.0 | 13 | 65.0 | 5 | 9.3 | 49 | 90.7 |

of the principal vertices, O, A and B, with control group learners significantly struggling on that aspect. Calculating $m_{AB}$ and $m_{BC}$ were challenging tasks for close to an average of seventy-nine percent of the participants, with control group learners dominating. Coordinates of C, D and E were only correctly identified by 4.1 percent (N=3) of the learners from the experimental group. By trial and error and without any mathematical evidence, 10.8 percent (N=8) got the correct coordinates of C, D and E. Figure OABCDE was successfully sketched in the Cartesian plane by only 5.4 percent (N=4) out of which one was from the control group.

## Significance of Mind Webs in Dealing with Selected Multiple-Concept Geometry Problems

In general, the majority of learners especially from the control group found it more challenging to come up with logical steps or mind webs. On average, sixty-five percent and ninety percent respectively, could not present any logical steps and mind webs. However, sixty percent (N=12) of the experimental learners came up with more coherent steps while twenty percent came up with (less detailed) mind webs, which some of them utilized as they worked along. There was a higher competence of concept integration by a few learners in the experimental group.

It also emerged that most learners could not utilize knowledge of the properties of various geometric figures in their pursuits to come up with answers. Most of them relied on non-geometric and mathematically incorrect methods to arrive at answers. For instance, one control group learner expressed that:

*To get surface area, multiply the length of the base of the right-angled triangle and the height of the triangle, and then add the product of the prism.*

*That is, Surface Area* $= bh + ph = 4 \text{x} 3 + 22 \text{ x } 3 = 12 + 60 = 80u^2$

*To get volume, multiply the length of the base with the height of the triangle. Multiply their product with the height of the prism. Then multiply the product of all sides by half:*

*Therefore,* $V = \frac{1}{2}bh.H = \frac{1}{2}(4 \times 3).3 = \frac{1}{2}(12).3 = 18u^3$

It is evident from the above presentation that such a learner is completely without knowledge and understanding of generally surface area or particularly how to get surface area of a triangular prism. However, the learner is aware of the volume formulae but faced the challenge of incorrect substitution of the dimension *H*, rendering the succeeding answer to be wrong. For both cases, incorrect statements were associated with incorrect formulae, working or answers.

Another common finding was the poor interpretation of gradient itself. At least fifty percent of the participants who correctly obtained $m_{BC}$ as $\frac{4}{3}$ incorrectly concluded that the gradient was synonymous with the $\frac{x}{y}$ relation, and thus translated the original points in reverse order by 4 units horizontally along the *x*-axis and 3 units vertically along the *y*-axis. The resultant coordinates for E (4;3) therefore rendered coordinates for D and C incorrect.

The majority of the respondents demonstrated poor understanding of the association between gradient and translation. One experimental learner categorically showed that comprehension and utilized it in finding all the coordinates of the other three 'tricky' vertices, C, D and E. Each step had the associated working immediately shown as follows:

*a (1) I would start by using the Pythagoras Theorem to find the unknown sides.*

*$Hyp^2 = Adj^2 + Opp^2$*

*$AB^2 = OA^2 + OB^2$*

*$AB^2 = 4^2 + 3^2$*

*Therefore, AB = $\sqrt{25}$= 5 units.*

*a (2) Since ABCD is a square, all sides are 5 units and since the two triangles are congruent they have the same lengths. To find the perimeter you add all the sides.*

*Therefore, Perimeter OABCDE = 4 + 3 + 5 + 5 + 5 + 5 + 3 + 5 = 39u.*

*a (3) Now to get the surface area you add the areas of all the 2D shapes that make up this 3D shape.*

*Therefore, Area $\Delta OAB = \frac{1}{2}bh = \frac{1}{2}.4.3 = 6u \times 2\Delta s = 12u^2$*

*Area □ ABCD = $s^2 = 5^2 = 25u^2$*

*Area ▭ OADE = $lb = 5.4 = 20u^2$*

*Area ▭ OBCE = $lb = 3.4 = 15u^2$*

*Therefore, Surface Area OABCDE = $(12 + 25 + 20 + 15)u^2 = 72u^2$*

*a (4) Now to get the volume, we use the formula $V = \frac{1}{2}bH = 6u^2 .5u = 30u^3$*

*b) Since O is the origin, then O(0;0) A is 4 units horizontally right along the x- axis, therefore its coordinates are A(4;0), and B is vertically upwards along the y- axis, therefore coordinates are B(0;3). Now to get the coordinates of C we use the gradient method. Understanding the relationship that BC ⊥BA, then $m_{BC} . m_{BA} = -1$. Hence,*

$$m_{BC} = -1 \times \frac{1}{m_{BA}} = -1 \times \frac{1}{-\frac{3}{4}} = \frac{4}{3}$$

*Parallel lines have the same gradient, $\frac{4}{3}$ therefore, a line parallel to BC has gradient hence if O(0;0), using $m_{BC} = \frac{\Delta y}{\Delta y}$ then $\frac{4}{3} = \frac{y_E - y_0}{x_E - x_0} = \frac{y_E - 0}{x_E - 0} = \frac{y_E}{x_E}$ equating gives $y_E = 4$ and $x_E = 3$, hence point E has coordinates E(3;4).*

*Therefore, using properties of rectangles and transformation geometry, coordinates of:*

*C(B(0;3) + (3;4)) = C(3;7) and*

*D(E(3;4) + (4;0)) = D(7;4)*

Figure 2 was the above experimental learner's graphical solution.

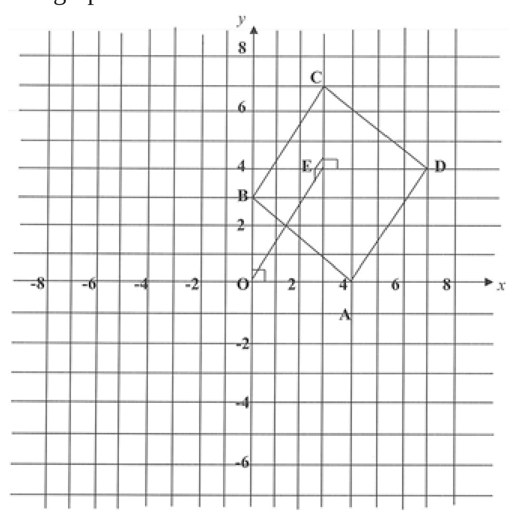


**Fig. 2. Experimental learner's graphical solution**

## DISCUSSION

It emerged from the study that the majority of the control group learners demonstrated significant incompetence in their ability to first present written steps or present a mind web then utilize the presented steps or mind web in their working. Nevertheless, most could neither mention nor employ the correct steps towards getting to the answers. In a separate study, Brinkmann (2003a) concludes that both mind mapping and concept mapping may be effective tools to improve mathematics achievement. Teachers would need to decide which of the two methods they would want to use depending on the outcomes they want to achieve.

Underperformance by the majority of learners could also imply that teachers were using teaching methods that do not promote understanding of the basic and important mathematical definitions and concepts. Teachers should be able to provide appropriate material taking cognizance of the cognitive levels of the learners they would be dealing with. This is consistent with Vinson's (2001) argument that using appropriate and concrete instructional materials is necessary to ensure that children understand mathematical concepts. Instructional practices to develop geometry understanding should encompass the range of learners' levels in that Grade. This means that the learners require multiple and diverse activities to enable them to progress through the levels of abstraction (Battista 2007).

Data also indicated that while most learners were not able to present their work in the form of a mind web or as written steps, there were higher tendencies to utilize the unmentioned concepts. However, it was revealed that learners who had presented their work in the form of mind webs or as written steps performed better. Twenty-one percent reasonably referred to their written steps or mind webs during their working. This is in sync with Brinkmann's (2003b) view that mind mapping assists mathematical thinking, helps students organize their knowledge and makes their knowledge structure visible. Mind mapping also fosters creativity, which has a positive effect on mathematical achievement.

In finding the hypotenuse (*H*) AB, it also emerged that no learner from the two groups was in a position to utilize or relate concepts of the equation of a circle with the Pythagoras Theorem , congruency and transformation geometry. With OA as $x = 4$ and AX as $y = 3$, using $4^2 + 3^2 = r^2$, the radius ($r$), say OX of a circle with center O(0;0) is 5u. It was important to understand that in the radius ($r$) is equal to the hypotenuse ($H$). Alternatively, with OA as $x = 4$ and AX as $y = 3$, detaching and superimposing in the Cartesian plane, then reflect it along the line $x = 2$ onto its image, say , one would then conclude that OX ($r$) = AB ($H$) = 5u and $\hat{A} = 90^0$. This possible shortfall by teachers to emphasize such aspects corroborates Brodie and Sanni's (2014:89) argument that teachers require 'specialized content knowledge' in order to do their work, which usually involves a deeper level of understanding of concepts, so that teachers can 'unpack' or understand learners' mathematical ideas.

Teaching learners to think beyond the obvious and in an abstract manner has the effect of enhancing positive attitudes towards learning and promotes better achievement in the subject. According to Stoker (2003:11), "learning is strongly and necessarily linked to teaching". This is supported by Van Hiele (1986) who states that the inability of many teachers to match instruction with their learners' levels of geometrical understanding is a contributing factor to their failure to promote meaningful understandings in this topic.

Teacher training colleges of mathematics and all subjects should promote the understanding of pedagogy so as to improve mathematics achievement in schools. Teachers should be trained to use creative and high quality instructional methods that train learners to understand, utilize and relate the various geometric concepts. This should be a normal classroom practice. Borko (2004) contends that the literature on teacher development argues strongly that professional development experiences need to be strongly related to the classroom. Similarly, Kazemi and Hubbard (2008) maintain that what teachers learn in professional development programs and what they learn in their practice interact and co-evolve both to further develop their knowledge of mathematics and of teaching and to produce knowledge that is useful in practice.

## CONCLUSION

The study concludes that the majority of learners taught through traditional methods have poor geometric concept formation and development as was indicated by their various poor approaches to the different sections of the questions compared to their counterparts. Thus, classroom practice had little emphasis on teaching and learning for geometric mastery.

Most learners exhibited poor time-management skills as evidenced sometimes by their rushed and poorly presented work. Teachers possibly did not put thrust on teaching leaners how to manage time during attempting given tasks.

Application of presumed geometric knowledge was not widespread among the participants. Most learners struggled to get the answers correct, as they could not manage to identify the integrating topics correctly. Linking measurement concepts to geometric concepts was a challenge for most learners. There were a few instances where learners could systematically and logically identify the concepts as they were to be developed in answering the questions.

Most learners could not come up with coherent mind webs despite that the phrase 'mind webs' consists of the English terms 'mind' and 'webs'. Thus integrating basically English and Art was also not applied in an effort to come up with general mind webs.

Current teaching methods promote poor retention, which is a factor with direct and negative effects on learners' potential to achieve better in mathematics. Teaching learners to deal with more abstract problems demands the lacking higher cognitive development approaches.

## RECOMMENDATIONS

Great emphasis on grasping basic mathematical concepts is fundamental in promoting high success rates in the subject. It should be pointed out that it is only through a clear understanding of the basics of a subject and full comprehension and application of pertinent concepts that one may be able to deduce meanings to most of the difficult and higher cognitive reasoning questions. Such an understanding will go a long way in aiding learners to get the grip of this very abstract subject.

Most learners do not avail themselves to the rigorous approaches and the reasoning required of mathematics. As most adopt the 'reluctant' approach when attempting mathematical problems, learners ought to individually apply reasonable effort and integrate other subjects with their learning in mathematics by implementing the minimum standards of self-regulation and motivation.

As teachers teach and as learners learn, choosing approaches that promote deep understanding of taught concepts ought to be part of the daily endeavors to raise performance. Math teachers should find different and interesting approaches to spice the traditional teaching methods of this abstract but important subject.

Government education and school authorities also need to recognize, credit and promote teachers who take initiatives and support them as far as they could bearing in mind that there are spillover effects to national development. Also, teaching time-management skills in dealing with given tasks should be promoted across curriculum subjects in schools.

Teachers are encouraged to try using the mind web teaching approach as it helps ascertain whether learners can present their ideas before answering. The approach also improves

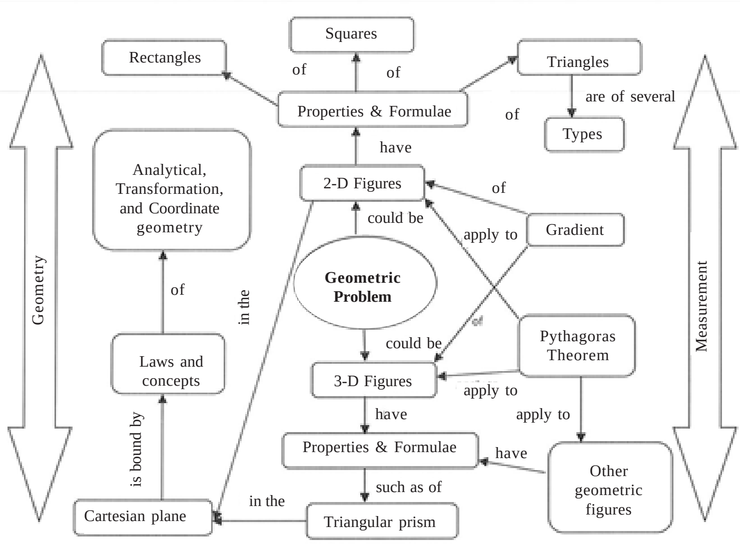


**Fig. 3. Kariyana and Sonn (2015) Geometric Problem Mind Web**

the command of learners' communication skills and understanding as well as utilization of mathematical terminology. Figure 3 is the researchers' proposed geometric problem mind web. It should be noted that mind webs differ depending on one's understanding of the question or topic.

According to Figure 3, geometric problems can be dealt with using geometric laws and concepts and/or measurement concepts. Teachers should teach learners to relate 2D and 3D figure(s) and consequently establishing their respective properties and formulae. Learners should be able to employ measurement concepts and/or apply the Pythagoras Theorem and gradient concepts to determine unknown lengths and angles. Alternatively, geometric laws and concepts may be utilized in relation to the properties of given geometric figures. Understanding the Cartesian plane and the coordinate system is also imperative. Recognizing that parallel and perpendicular lines, their meaning/definition, examples, drawing, significance, shapes, properties and so on, are mainly measurement properties, and the ability to apply that in geometry in calculating gradient, linking with known measurement concepts is crucial. Understanding congruency might also be an important aspect to help get the coordinates. It is impossible to separate these two integrated topics if significant understanding for dealing with related multiple-concept, higher-order geometry problems is expected from learners.

The researchers argue that compromised concept formation tends to compromise concept development and hence integration, therefore, teachers ought to consider integrating their mathematics lessons with this approach in attempts to improve and spice classroom effectiveness and overall mathematics attainment.

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