PRINT: ISSN 0975-1122 ONLINE: 2456-6322
Int J Edu Sci, 12(1): 29-37 (2016)
DOI: 10.31901/24566322.2016/12.01.04

The Nature of Middle School Mathematics Teachers' Subject Matter Knowledge: The Case of Volume of Prisms[1]

Reyhan Tekin-Sitrava[1] and Mine Isiksal-Bostan[2]

[1]Hacettepe University, Ankara, Turkey
[2]Middle East Technical University, Ankara, Turkey

KEYWORDS Alternative Solution Methods. Middle School. Mathematics Teachers. Prisms. Teacher Knowledge. Volume

ABSTRACT This qualitative case study examined Turkish middle school teachers' subject matter knowledge in terms of alternative solution methods that they propose to calculate the volume of prisms. The data was collected from four middle school teachers through the questionnaire, semi-structured interviews, and classroom observations. The results from the questionnaire revealed that the middle school teachers stated four alternative solution methods; volume formula, systematic counting, layer counting and column/row iteration. However, analyses of classroom observations showed that they only used the volume formula to calculate the volume of prisms in their lessons. Furthermore, it could be concluded that the variety of alternative solution methods changed with respect to the way that the researchers asked the questions even if all the questions were related to calculating the volume of prisms. More specifically, if the question was a little complicated and the teachers were not familiar with it, then the teachers focused on using the volume formula.

INTRODUCTION

Teacher knowledge is one of the key components for effective teaching that affect student achievement (Kleickmann et al. 2013; Ball et al. 2008). Primarily, Shulman (1986) expressed the complexities of the major categories of teachers' knowledge: subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curricular knowledge (CK). Among these categories, Shulman's (1986) SMK involves knowing the facts, truths and concepts, comprehending the concepts and principles, explaining the reasons for learning these concepts, and relating the concepts within and without the discipline. By virtue of the importance of SMK for effective teaching, many researchers have focused on investigation of mathematics teachers' SMK in terms of several dimensions. Among the dimensions of Shulman's SMK, teachers' knowledge of generating alternative solution methods for solving the questions was taken as the dimension to investigate middle school mathematics teachers' SMK in the current study. While generating alternative solution methods, teachers will represent their mathematical ideas, provide mathematical explanations and comprehend the concepts and principles regarding any topic (Shulman 1986).Teachers' SMK alone was not enough to make the subjects more comprehensible for students (Maxedon 2003). It is necessary to have deep geometry knowledge due to the fact that geometry has crucial role in teaching and learning in other subject areas of the mathematics curriculum and other disciplines (National Council of Teachers of Mathematics [NCTM], 2000). Since geometry is connected with mathematics (Jones 2002) and it has an outstanding role in the interpretation and reflection on our physical environment, it helps students to develop their critical thinking, spatial visualization, reasoning, proof and problem solving skills (Jones 2002). Because of these aspects, NCTM (2000) stated that geometry benefits to both teachers and students in teaching and learning mathematics and other disciplines. However, The International Commission on Mathematical Instruction [ICMI] (1995) stated that the teaching of geometry is complex endeavor and there are many obstacles to teaching of geometry. In accordance with the results of several studies, one of the most important obstacles is inadequate teachers' knowledge on teaching of geometry (Aslan-Tutak and Adams 2015; Esen and Cakiroglu 2012; Gomes 2011). In contrast to importance of geometry for mathematics teaching, many researchers did not focus on investigating teachers' SMK on geome-

Address for correspondence:
Reyhan Tekin-Sitrava
Hacettepe University, Faculty of Education
Department of Elementary Education,
Ankara, Turkey
E-mail: reyhan_tekin@yahoo.com

try topics. As Maxedon (2003) stated, teachers' knowledge of geometry has important role for understanding teachers' knowledge of mathematics, such researches would also extend the understanding of the teachers' mathematics knowledge.

Starting from this analysis, it was noticed that there is a need for a research study, whose aim is to present teachers' knowledge on geometry topics. In order to satisfy this need, the current study aims to explore teachers' knowledge related to a specific topic in geometry.

Purpose of the Study

Among the topics of geometry, ICMI (1995) reported that "three-dimensional geometry has almost disappeared or has been confined to a marginal role in the curricula in most countries" (p.91). Similarly, teachers' SMK on three-dimensional geometry has not been investigated yet in detail within the literature. When low students' performance on the three-dimensional geometry (Tekin-Sitrava and Isiksal-Bostan 2014; Olkun 1999; Battista and Clements 1996; Ben-Chaim et al. 1985) and the relationship between students' learning and teachers' knowledge are taken into consideration, it is indispensable to investigate teachers' SMK related to the three-dimensional geometry. From this point of view, the aim of this study is to investigate four Turkish middle school mathematics teachers' knowledge regarding the volume of prisms. As it was stated, in the current study, teachers' knowledge is investigated from the point of different dimension that is knowledge of generating alternative solution methods. By this way, it is hoped to complete the missing part of the picture of teachers' knowledge literature.

Moreover, the significance of the current study is also rooted in the data collected from a real classroom context in Turkey. Due to the fact that data was collected via classroom observation, the study provides concrete examples of teachers' knowledge in relation to a specific topic, which was calculating the volume of prisms. Besides, the present study investigates experienced teachers' knowledge. In the related literature, research studies have generally focused on knowledge of pre-service teachers (Aslan-Tutak and Adams 2015; Bozkurt and Koc 2012; Contreras et al. 2011; Huang and Kulm 2012). However, pre-service and novice teachers generally may not have a robust SMK. Therefore, it is expected that valuable information will be gained concerning how teachers use their SMK for effective teaching.

Due to these reasons, this study aims to answer the following question:

What are the alternative solution methods that four experienced Turkish middle school mathematics teachers propose to calculate the volume of prisms?

METHODOLOGY

Design of the Study

In this research study, the aim was to gain an in-depth understanding of the middle school teachers' SMK in terms of the knowledge of the alternative solution methods that they propose to calculate the volume of prisms. In order to achieve the purpose of the study, qualitative case study design was used in which Turkish middle school teachers constitute the case of the study. The current study was conducted in a real context. Therefore, in this study, the middle school teachers' SMK was analyzed within the context of their school and the classroom in which they teach. The teachers worked in different public schools in Turkey which were close to each other. There were 400 students in each school and 20-25 students in each class, which was observed. The students in all schools had 4 hour-mathematics lesson every week. Moreover, Turkish mathematics curriculum which was renewed in 2004 was being implemented in each school. With the curriculum, the students adopt the role of being active, researching, constructing their knowledge, solving problems and sharing their solutions with their classmates. In terms of teachers' role, the curriculum highlights the importance of creating classroom environment where teaching the topics with applying different instructional methods and activities, using manipulative and technology, and giving students opportunities to be more active (Ministry of National Education [MoNE] 2006).

Participants

Data were collected from four Turkish middle school mathematics teachers who were called as Mrs. Erkan, Mrs. Ocak, Mr. Atak and Mrs. Aksoy in the current study. While Mr. Atak had

12 years teaching experience, others had at least 30 years teaching experience. In other words, Mrs. Erkan had 31 years, and Mrs. Ocak and Mrs. Aksoy had 32 years teaching experience. Experienced teachers were chosen as subjects of the study with the following reasons: First they have been teaching the topic, the national curriculum determines that the volume of prism is 8th grade topic. Second, parallel to their roles stated in the mathematics curriculum, they tried to enrich their lessons with different representations, using manipulative and technology, encourage students to make presentations, prepare posters, etc., and link mathematics with daily life.

Measuring Tools

To gather and triangulate data, the following data collection tools were used: (1) A questionnaire on calculation the volume of prism; (2) interviews following the questionnaire; (3) classroom observations.

In order to understand the middle school teachers' SMK in terms of alternative solution methods that they propose to calculate the volume of prism, the questionnaire consisted of 2 open-ended structured questions was used. It was prepared by the researchers based on the related literature (Battista and Clements 1996; Ng 1998) in accordance with on the subjects covered in the Turkish middle school mathematics curriculum. The questionnaire is provided in Figure 1.

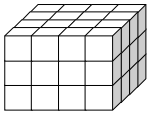
Both questions focused on calculating the number of unit cubes which constitute the square prism and the teachers were expected to solve them using variety of solution methods. Although it was asked to directly calculate the volume of prism in the 1st question, the 2nd question consisted of two phases, removing the one layer from all faces of the prism and then calculating the remained number of unit cubes. Due to the fact that the content of the questions were different, it was expected the teachers to generate different solution methods to solve them.

After administering the questionnaire, semi-structured interviews were conducted with 4 middle school teachers in order to get in-depth exploration on their SMK. During the interviews, the middle school teachers explained their solution methods in detail. According to participants' responses, additional questions were asked as well. Example interview questions asked were provided in Appendix.

Although interviews are the most commonly used data collection tool in qualitative research, Merriam (1998) stated that "observational data represents a firsthand encounter with the phenomenon of interest rather than a secondhand account of the world obtained in the interview" (p.94). Therefore, to obtain a complete picture of the teachers' SMK on the alternative solution methods to calculate the volume of prism, their teaching on the subject in the classroom was observed. During the observations, the important point that was the solution methods that the teachers use while

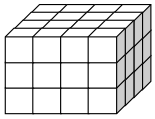
Fig. 1. Questionnaire

Question 1.



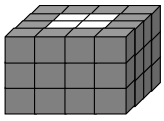
Write down all the methods that you know which could be used to calculate the number of unit cubes in the square prism.

Figure A



Question 2.

Figure B



Write down all the methods that you know which could be used to calculate the number of unit cubes remain when one layer of unit cubes is removed, as indicated with grey color in Figure B, from the all faces of the square prism which is presented in Figure A?

teaching the topic. All observations and the interviews were video-taped with the permission of the participants.

Data Analysis

In this study, a constant comparative method of data analysis was utilized. First, the codes and categories derived from the literature (Battista and Clements 1996), the participants' statements, and our experiences with the data were identified. Then the categories were integrated based on their properties until the categories reached saturation. In the next step of the analysis, the consistency among the categories and within their properties was investigated. Lastly, the categories, which were labeled based on the participants' statements and the related literature, were integrated to create the themes. In order to ensure validity and reliability of the study, the mathematics educators discussed the codes, and the data was coded by two coders. Later, their codes were compared and discussed until reaching a total consensus. At the end of this process, a final coding scheme was created and presented in Table 1.

Table 1: Knowledge of alternative solution methods

Coding	*Meaning*
Volume Formula	Multiplying the depth, the width and the height of the prism Multiplying the lengths of three edges Multiplying area of the base of 3D solids by its height
Systematic Counting	Counting cubes systematically, attempting to count both inside and outside cubes. He or she might, for instance, count the cubes on all the outside faces, and then attempt to determine how many are in the center
Layer Counting	Counting the number of unit cubes in one layer, and then multiplies this number by the number of layers or uses addition to obtain the total
Column/Row Iteration	Counting the number of cubes in one row or column and uses skip-counting

In the next section, the main categories from the questionnaire, interviews, and video transcripts are explained while presenting the findings of the study.

FINDINGS

In this study, the aim was to examine four middle school teachers' knowledge of alternative solution methods that they develop to calculate the volume of prisms. The analysis of the data obtained from the questionnaire, semi-structured interviews, and classroom observations revealed that the four middle school teachers proposed the following four alternative solution methods; *volume formula, systematic counting, layer counting and column/row iteration*. The alternative solution methods that middle school teachers stated for the each question were summarized in Table 2.

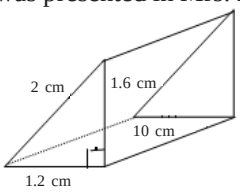
Volume Formula

One of the solution methods that the middle school teachers proposed to calculate the volume of prisms was *volume formula*. Battista and Clements (1996) defined volume formula as multiplying the depth, the width and the height of the prism. Based on the analysis of the data, all the middle school teachers defined volume formula in two ways: 1) multiplying the lengths of three edges, 2) multiplying the area of the base of the prism by its height.

As shown in Table 2, four teachers proposed volume formula to calculate the volume of prisms. As an example, Mrs. Erkan's explanation for the question 1 was presented below.

The bases and the height, namely the volume could be calculated with the volume formula. How is that done? It can be achieved by counting the number of unit cubes in the height and in the edges of bases or by finding the volume from a x b x c. Or by saying the base is a rectangle and [you] multiply the base with the height [of the prism] to answer the question.

Based on the data gathered from classroom observation, all the teachers frequently used the volume formula method to calculate the volume of prisms in their lessons. An example related to using the volume formula method in the lesson was presented in Mrs. Erkan's lesson below:



What is the volume of chock whose shape is right triangle prism in the figure?

Table 2: The solution methods proposed by the middle school teachers

	Mrs. Erkan	*Mrs. Ocak*	*Mr. Atak*	*Mrs. Aksoy*
Question 1	• Volume formula • Layer counting • Coloum/Row ineration	• Volume formula • Systematic counting	• Volume formula • Systematic counting	• Volume formula • Layer counting • Coloum/Row ineration
Question 2	• Volume formula	• Volume formula	• Systematic counting	• Could not develop any correct method

Mrs. Erkan explained that to calculate the volume of a prism, the base area of the prism and its height are multiplied. Then she clarified that the base of the prism is a right triangle with right edges of 1.2 cm and 1.6 cm and the height of the prism is 10 cm. Then she solved the problem as follows:

V= the base area x the height

$V = \frac{1.2 \times 1.6}{2}.4$

$V = 0.96 \times 4$

$V = 3.84\ cm^3$

Similar to Mrs. Erkan, other teachers applied volume formula frequently in their lessons.

Systematic Counting

The analysis of the data revealed that 2 of middle school teachers proposed *systematic counting method* as an alternative method to calculate the volume of prisms (Table 2). Battista and Clements (1996) defined the method as "students counts cubes systematically, attempting to count both inside and outside cubes. He or she might, for instance, count the cubes on all the outside faces, and then attempt to determine how many are in the center." (p. 263).

Mrs. Ocak and Mr. Atak emphasized the systematic counting method to calculate the volume of prisms. Both teachers explained that if the prism was presented as in the question 1, the unit cubes might be counted on both inside and outside of the prism. Similarly, Mr. Atak specified this method for the solution of question 2 as presented below:

Now, do we count the cubes on the outer faces? If we count them, here [the length of the width, depth and height] lessened. The reminder of the cubes constitutes a rectangular prism. The rest of the unit cubes could be counted one by one.

Although 2 teachers generated systematic counting method to calculate the volume of prism, none of the teachers use this method in their lessons.

Layer Counting

The analysis revealed that middle school teachers stated *layer counting method* as another alternative solution method to calculate the volume. Similar to the volume formula and systematic counting methods, Battista and Clements (1996) explained layer counting as "the student conceptualizes the set of cubes as forming a rectangular array organized into layers." (p. 263). In other words, layer counting means counting the number of unit cubes in one layer, and then multiplying this number by the number of layers or using addition to obtain the total.

As shown in Table 2, only two participants (Mrs. Erkan and Mrs. Aksoy) proposed layer counting method to solve question 1. Mrs. Erkan's explanation is presented below.

I thought that there are 16 unit cubes at the bottom layer and there are 3 layers. For this reason, the total could be calculated as 16+16+16.

Mrs. Erkan used this method to solve a problem in her lesson. She asked the students how they would calculate the number of unit cubes in the rectangular prism given in Figure 2.

Students proposed to use the volume formula method. Then Mrs. Erkan showed her students different methods to calculate the number of unit cubes. One of her method is given in Figure 3:

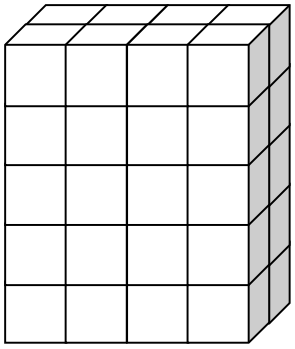


Fig. 2. An example from Mrs. Erkan's lesson

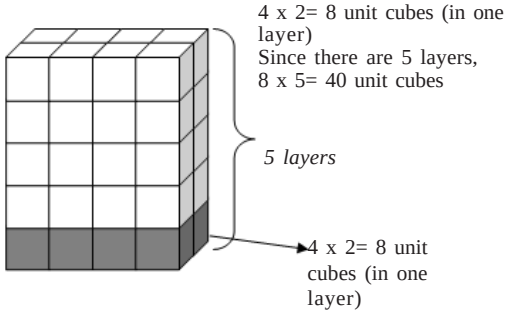


Fig. 3. An example of using layer counting from Mrs. Erkan's lesson

On the other hand, although Mrs. Aksoy referred to layer counting method during the interview, she did not use this method in her lesson. However, Mrs. Ocak and Mr. Atak did not explain the layer counting method during their lessons or the interviews.

Column/Row Iteration

The analysis of the data showed that middle school teachers proposed *column/row iteration method* as an alternative solution method to calculate the volume of prisms. Similar to other methods, this was also defined by Battista and Clements (1996) as "students count the number of cubes in one row or column and use skip-counting (pointing to successive rows or columns) to get total" (p.263).

Similar to the layer counting method, Mrs. Erkan and Mrs. Aksoy emphasized the use of the column/row iteration method only for question 1 (Table 2). Below is the related vignette from Mrs. Aksoy's interview.

The number of unit cubes in each row might be counted. Namely, I thought that there are 4 unit cubes in each row (indicated in the Figure 4 with grey color). How many rows are there? 12 rows; 4 x 12= 48.

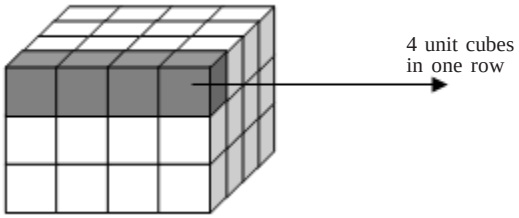


Fig. 4. Mrs. Aksoy's explanation of example of using column/row iteration

Another example of using column/row iteration was explained by Mrs. Erkan as shown in Figure 5:

There were 3 unit cubes here, 3 here, 3 here…[one is presented in the Figure 5 with grey color]. Namely, it occurs by counting. It was not 1,2,3; by counting 3 by 3

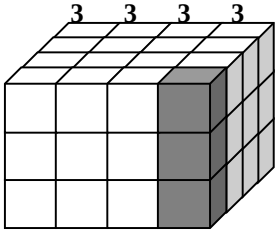


Fig. 5. Mrs. Erkan's explanation of example of using column/row iteration

Additionally, Mrs. Erkan focused on this method to solve the question that she presented in her lessons. As mentioned before, Mrs. Erkan asked her students the number of unit cubes in the rectangular prism given in Figure 2. As stated above, the students solved the problem using a volume formula, then Mrs. Erkan solved it using the layer counting method. Moreover, Mrs. Erkan solved the problem using the column/row iteration method as follows:

Mrs. Erkan wrote the number of unit cubes in each column by adding the previous one to obtain the total.

As with the layer counting method, Mrs. Ocak and Mr. Atak did not refer to this method to calculate the volume of prisms (Table 2).

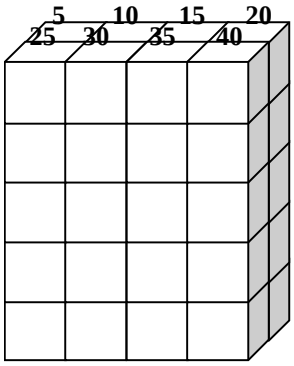


Fig. 6. An example of using column/row iteration from Mrs. Erkan's lesson

To summarize the analysis of the data, it showed that the four middle school teachers proposed the four following methods; volume formula, systematic counting, layer counting and column/row iteration. Although the first method was indicated as a way of solving the problem in the question 1 by all the teachers, systematic counting was denoted by Mrs. Ocak and Mr. Atak to solve questions 1 and 2. Layer counting and column/row iteration methods were only specified by Mrs. Erkan and Mrs. Aksoy for solving question 1 (Fig. 6, Table 2).

DISCUSSION

Generating alternative solution methods to the given questions is one of the dimensions of teachers' SMK that formed the basis of this research study. The data analysis revealed that middle school teachers could generate four alternative solution methods to calculate the volume of prisms. Of these methods, systematic counting, layer counting and column/row iteration were accepted as more complicated than the volume formula. To generate these three methods, it is necessary to know more than the multiplication of three edges (Battista and Clements 1996). For instance, to use layer counting method, people should realize that a prism is composed of the layers. Similarly, to use column/row iteration method, it is necessary to know that a prism formed by columns or rows (Tekin-Sitrava and Isiksal-Bostan 2014). Based on the analysis of the data gathered from classroom observations, the teachers did not use the solution methods, systematic counting, layer counting and column/row iteration, during teaching the volume of prisms in their lessons. They only used the volume formula to calculate the volume of prisms. This result is consistent with the previous studies which concluded that teachers relied on the usage of formula (Murphy 2012; Rizvi 2004). The reason for using volume formula might be the teachers' limited understanding of the mathematical concepts (Berenson et al. 1997). Since the teachers had limited understanding of the topic, they would not explain the alternative solution methods (Murphy 2012; Simsek et al. 2012), such as layer counting method, to their students and furthermore, they might not encourage students to use these methods even if they could generate them. As a result, it could be concluded that middle school teachers, who focused on using formula, probably had a limited understanding regarding the topic and were reliant on formulas in their teaching.

Another important finding of this study is related to the variety of alternative solution methods that middle school teachers generate. More specifically, based on the analysis of the data, the variety of alternative solution methods changed with respect to the way that the researchers asked the questions even if all the questions were related to calculating the volume of prisms. Two questions were given to the teachers to generate alternative solution methods to calculate the volume of prisms. In one of the questions, teachers were expected to directly calculate the number of unit cubes of the prism (Question 1). There was no extra challenge in terms of solving this question and all the middle school teachers were able to develop at least two solution methods. This result could be interpreted as the teachers could develop more than one strategy to solve simple questions. In other words, the middle school teachers did not need broad SMK to use alternative solution methods if the volume of prisms was asked directly. However, in another question, the teachers were asked to calculate the number of unit cubes of prism when one layer of unit cubes is removed from all faces of prism (Question 2). Since this question has two phases, removing the outer faces and then calculating the remained number of unit cubes, this question is more complicated than the previous one. It was found that the middle school teachers applied only one solution method to solve this question. Two mid-

dle school teachers used the volume formula, one applied systematic counting method however, one teacher was unable to solve the question when one layer of unit cubes is removed from all the faces of the prism. This result reveals that if the question was a little complicated and the teachers were not familiar with it, then the teachers focused on using the volume formula. The reason for this might be that teachers had limited knowledge of generating alternative solution methods to solve complicated questions.

To sum up, teachers could develop alternative solution methods to calculate the volume of prism during the interviews. However, they did not use these methods in their lessons. The reason for this contradiction might be that the middle school teachers lacked a deep knowledge of the subject matter. As Lederman, Gess-Newsome and Latz (1994) explained, teachers' low level of SMK influences their instructional decisions. Therefore, the teachers would not be effective in developing children's understanding of a topic by providing them different solution methods (Murphy 2012). Due to the fact that the middle school teachers did not provide alternative solution methods to their students while they were teaching the topic, it could be concluded that their SMK was inadequate in terms of knowledge of alternative solution methods.

CONCLUSION

The current study verified the argument in the literature that teachers has no deep and sufficient SMK. Teachers need to recognize that just knowing and applying procedures or formulas does not mean that they have a deep understanding of SMK for teaching mathematics. In the current study, the middle school teachers had limited SMK in terms of generating alternative solution methods. Generally, they did not develop alternative solution methods to calculate the volume of prisms while teaching the topic during their lessons. Thus, the middle school teachers might enroll in professional development programs to help them develop their understanding of the volume of prisms and enrich their knowledge of alternative solution methods which could be used to solve the questions related to the volume of the prisms. The middle school teachers in the current study had not participated in any professional development programs. However, participating in professional development programs has several benefits for the teachers such as; providing opportunities to develop the depth and breadth of their mathematics content knowledge. In such programs, middle school teachers might study on developing alternative solution methods to solve variety of questions related to the volume of prisms. Accordingly, they might share and discuss these methods with other teachers.

RECOMMENDATIONS

This study was a case study focusing on only one dimension of teachers' SMK that is generating alternative solution methods to calculate the volume of prisms. Therefore, further research studies should be carried out to identify teachers' SMK on other dimensions such as explaining the reasons for learning and teaching the volume of prisms, relating the topic within mathematics and other disciplines, representing mathematical ideas, and providing mathematical explanations and procedures with their justifications. This type of research would provide a larger picture of teachers' SMK regarding the volume of prisms. Moreover, future studies could be conducted to investigate teachers' SMK about the other important areas of geometry, such as geometric figures and their characteristics and relation between them, coordinate geometry, and geometry transforms. This would allow researchers to display the complete picture of middle school teachers' knowledge of geometry.

NOTE

[1] This research is the part of the first author's dissertation study.

REFERENCES

Aslan-Tutak F, Adams TL 2015. A study of geometry content knowledge of elementary preservice teachers. *International Electronic Journal Environmental Education*, 7(3): 301-318.

Ball DL, Thames MH, Phelps G 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5): 389-407.

Battista MT, Clements DH 1996. Students' understanding of three-dimensional rectangular arrays of cubes. *Journal for Research in Mathematics Education*, 27 (3): 258-292.

Ben-Chaim D, Lappan G, Houang RT 1985. Visualizing rectangular solids made of small cubes: Analyzing and affecting students' performance. *Educational Studies in Mathematics*, 16: 389-409.

Berenson S, Von Der Valk T, Oldham E, Runesson U, Moreira Q, Broekman H 1997. An international study to investigate prospective teachers' content knowledge of the area concept. *European Journal of Teacher Education,* 20(2): 137-150.

Bozkurt A, Koc Y 2012. Investigating first year elementary mathematics teacher education students' knowledge of prism. *Educational Sciences: Theory and Practice*, 12(4): 2949-2952.

Contreras JM, Batanero C, Díaz C, Fernandes JA 2011. Prospective Teachers' Common and Specialized Knowledge in a Probability Task. *Paper presented at the Seventh Congress of the European Society for Research in Mathematics Education* in University of Rzeszów, Poland, February 9 to 13, 2011.

Esen Y, Cakiroglu E 2012. Ilkögretim matematik ögretmen adaylarinin hacim ölçmede birim kullanmaya yönelik kavrayislari. *Matematik Egitimi Dergisi*, 1: 21-30.

Gomes A 2011. Portuguese pre-service elementary teachers' knowledge of geometric transformations: An exploratory study. In: C Smith (Ed.): *Proceedings of the British Society for Research into Learning Mathematics*, 31(3): 59-64.

Huang R, Kulm G 2012. Prospective middle grade mathematics teachers' knowledge of algebra for teaching. *The Journal of Mathematical Behavior,* 31(4): 417-430.

Jones K 2002. Issues in the teaching and learning of geometry. In: Linda Haggarty (Ed.): *Aspects of Teaching Secondary Mathematics: Perspectives on Practice*. London: Routledge Falmer, pp. 121-139.

Kleickmann T, Richter D, Kunter M, Elsner J, Besser M, Krauss S, Baumert J 2013. Teachers' content knowledge and pedagogical content knowledge. The role of structural differences in teacher education. *Journal of Teacher Education*, 64(1): 90-106.

Lederman N, Gess-Newsome J, Latz M 1994. The nature and development of pre-service science teachers' conceptions of subject matter and pedagogy. *Journal of Research in Science Teaching*, 31: 129-146.

Maxedon SJ 2003. *Early Childhood Teachers' Content and Pedagogical Knowledge of Geometry.* PhD Thesis, Unpublished. USA: The University of Arizona.

Merriam SB 1998. *Qualitative Research and Case Study Applications in Education.* Revised Edition. San Francisco, CA: Jossey-Bass.

Ministry of National Education [MoNE] 2006. *Ilkögretim Matematik Dersi Ogretim Programi 6-8. Siniflar: Ögretim Programý ve Kilavuzu.* Ankara, Turkey: MoNE.

Murphy C 2012. The role of subject knowledge in primary prospective teachers' approaches to teaching the topic of area. *Journal of Mathematics Teacher Education,* 15:187–206.

National Council of Teachers of Mathematics [NCTM] 2000. *Principles and Standards for School Mathematics*. Reston, VA: Author.

Ng GL 1998. *Exploring Children's Geometrical Thinking*. PhD Thesis, Unpublished. Norman: University of Oklahoma.

Olkun S 1999. *Stimulating Children's Understanding of Rectangular Solids Made of Small Cubes.* PhD Thesis, Unpublished. USA: Arizona State University.

Rizvi NF 2004. *Prospective Teachers' Knowledge about the Concept of Division*. Bedford Park, South Australia: Flinders University of South Australia

Shulman LS 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher,* 15(2): 4-14.

Simsek A, Ozkaya M, Gedik SD, Senel EO, Konyalioglu AC 2012. An Investigation on Geometry Content Knowledge of Prospective Mathematics Teachers. *11th Mathematics Symposium: Mathematics in Anatolia*, Samsun, September 19 to 21, 2012.

Tekin-Sitrava R, Isiksal-Bostan M 2014. An investigation into the performance, solution strategies and difficulties in middle school students' calculation of the volume of a rectangular prism. *International Journal for Mathematics Teaching and Learning*, 1-27.

The International Commission on Mathematical Instruction [ICMI] 1995. Perspectives on the teaching of geometry for the 21st century. *Educational Studies in Mathematics,* 28(1): 91-98.

Paper received for publication on July 2015
Paper accepted for publication on December 2015