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Exploring Learner Errors in Solving Quadratic Equations

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ABSTRACT The study focuses on exploring grade 11 learner errors and misconceptions in solving quadratic equations at a secondary school in Gauteng Province in South Africa. The study identified learners' errors and misconceptions in solving quadratic equations and investigated why learners made those errors. Both qualitative and quantitative designs were used in the study. To collect data, learners solved quadratic equations tasks by factoring, completing the square and using the quadratic formula. Thereafter, semi-structured interviews were conducted with six learners. The selection of these learners depended on the type of errors shown on their scripts. The findings of the study revealed that the learners' lack algebraic competency encumbered their solutions to quadratic equations. The researchers recommend that more research be undertaken to determine the impact of teaching when learners' identified errors and misconceptions in solving quadratic equations shown in this study are used as instructional resources.

INTRODUCTION

South Africa like other countries has a clear goal of providing quality education to all the young people in the country. The South African government through the Department of Education has a responsibility to make science and mathematics education available and accessible to every South African irrespective of gender, social class, ethnic origin and race.

Science and mathematics education are the cornerstone for the country's economic development (Mhakure et al. 2014). Having a solid background in mathematics serves as a gateway to future professions in a variety of fields like medicine and engineering. A thorough understanding of mathematics is an asset, essential for applicants interested in obtaining better employment the world over (Bybee 2015). In South African public schools, classrooms are still crowded, and the ratio of teacher to learner is 1:32. The teacher-learner ratio in most of these public schools is high. This crowdedness in the classrooms is a problem since little attention is given to each learner for them to learn mathematics and science. The Dinaledi program aims at reducing learner-teacher ratios in the classrooms enabling teachers to pay more attention to learners. To facilitate this, the Department of Basic Education introduced additional teacher posts in mathematics and science that are over and above the normal post provision. In some schools, two teachers share and teach the same class. Yet this strategy is not enough as students continue to underachieve in mathematics (Taylor and Taylor 2013).

Poor learner mathematical performance in South Africa is well known in academic circles (Taylor and Taylor 2013). Umalusi, the examinations board, yearly reports on frequent learner mathematical errors in examinations. One area where the learners' performance is very poor is algebra. Handling algebra, including solving quadratic equations remains problematic for learners (Makonye and Khanyile 2015). One of the researchers as an educator and a marker in the NSC examinations developed a need to study the learners' errors and misconceptions in quadratic equations to try to find the reasons why learners have difficulties in this topic. The teachers' understanding of learners' challenges helps them deliver more effective lessons (Riccomini 2005). The researchers hope that the findings from this study might lead to the improvement of teaching strategies in this topic for the benefit of the struggling learners.

Objectives

The purpose of the study is to identify errors learners make in solving quadratic equations and to analyze how these emerge from their misconceptions.

Literature Review

The theoretical framework of the study is informed by sociocultural learning theory (Vygotsky 1986). This theoretical framework informs how and why learners interpret mathematics in ways that result in them making errors and misconceptions (Makonye 2011).

The sociocultural learning theory by Vygotsky (1986) emphasizes the interdependence of social and individual processes in the construction of knowledge and understands how individuals learn to think as a consequence of their activity in social transactions. It focuses on how learners construct their own knowledge through social cultural interaction. Learners' errors can be understood in this sociocultural theory of learning in that teachers who themselves have many errors and misconceptions in mathematics can transfer them to their learners since the theory suggests that the child will learn when encouraged to with an adult's assistance (Mamba 2012). The misconceptions can be communicated to learners by their teachers or the learners misinterpret what the teacher communicated to them since learning takes place through social interaction. This means that teachers need to be more knowledgeable than the learners so that appropriate mathematical concepts are taught to learners to avoid misconceptions to be communicated during teaching and learning.

One of the useful principles in Vygotsky's learning theory is the concept of the Zone of Proximal Development (ZPD). This is the gap between the child's capability to solve problems on his own and his capability to solve them with support. The ZPD is not a discreet zone, which an individual possesses at a particular stage of their development, but it is a process that can occur at any stage of development. ZPD believes that learning should be a joint activity between the teacher and the learner based on mutual co-operation and agreement (Osei 2006). This means that teachers must be prepared to help learners when they are stuck and cannot proceed with the solving of mathematical problems to minimize their errors and misconceptions. Learners cannot fully learn alone and they need assistance from a more knowledgeable person like a teacher.

Mathematical Errors and Misconceptions

According to Riccomini (2005), unsystematic errors are slips that learners can correct by themselves without outside help. Learners can rectify their errors without any assistance or without being told that they are wrong. According to Riccomini (2005), these unsystematic errors are random errors, which occur with no evidence of a recurring incorrect way of thinking. On the other hand, systematic errors recur over and over again due to a mistaken line of thinking (Green et al. 2008). According to Olivier (1989), misconceptions are instruction resistant. Some errors keep on coming up no matter how good learners are taught. Misconceptions are not terrible things to be uprooted but making errors is part of learning. These are a result of the learners' attempts to construct their own knowledge. Teachers need to keep on giving learners feedback after an activity so that errors and misconceptions are constantly rectified.

Luneta and Makonye (2010) propose that mathematical errors originate from lack of procedural and conceptual understanding (Hiebert and Levefre 1986). These are as follows.

- Conceptual errors occur when learners fail to understand the concepts involved in a given task or when learners fail connect the relationships between concepts.
- Procedural errors are errors displayed by learners when they cannot use an algorithm properly.
- Arbitrary errors occur when the learners change the questions to suit their level of understanding.

MATERIAL AND METHODS

The research design the researchers used is a mixed methods approach. This employs a mixture of quantitative and qualitative research (Merriam 1998). Quantitative data was collected from the quadratic task with an intention of identifying learners' errors and misconception in quadratic equations. Qualitative data was gathered via interviews with an aim of finding possible reasons why learners make those errors. The researchers assumed that collecting diverse types of data could provide a clearer understanding of the learner errors and misconceptions on quadratic tasks. The design was employed mainly because of its strengths since the merits of the approaches compliment each other (Opie 2004).

While the quantitative part helps quantify the prevalence of learner error types and misconceptions, qualitative methodology suited this study as it gives a detailed explanation of how learners reason to form errors in solving a mathematical problem (Merriam 1998). The qualitative approach helps the researchers understand participant learners' views about errors displayed in their quadratic task scripts. It offers an interpretation of how learners conceive or misconceive mathematical concepts resulting in errors and misconceptions. The researchers chose qualitative methodology because it is concerned with descriptions of thinking shown by learners' answers to specific questions. Here the researchers wanted to understand the thinking processes that led to learners making errors in solving quadratic equations. This assisted the researchers in understanding the learners' meanings and intentions when their productions display errors and misconceptions. Qualitative analysis focused on the most prominent errors and took the lead over quantitative method in this study.

Sample

Maree (2007:79) defines a sample as, "the selected portion of the population for study". The sample for study was a mixed ability group of 12 boys and 20 girls from a grade 11 Johannesburg high school class, South Africa in May 2013. Sampling of participants involved in this study used a combination of purposive and convenience sampling strategies. In convenience sampling, the most accessible participants are chosen while in purposive sampling a deliberate selection of participants that meet criteria relevant to the research study are chosen (Cohen and Marion 1995). The study also allowed for convenience and purposeful sampling of participants because of their accessibility to the researchers and their willingness and cooperation to participate. The learner participant also met the criteria relevant to the study since they were selected on the basis that they are mathematics learners.

Procedure for Data Collection

Quadratic tasks were administered to learners to write in their class for one hour. For anonymity purposes, learners used fake names but indicated their class list number on their scripts. After marking the task, the scores per question were recorded against the correct answer for all the participants. Also, the scripts were sorted out and grouped, putting scripts with similar errors together. The researchers named the errors according to how they were produced. The researchers also counted the number of errors depending on the errors type displayed by learners to build frequency table of learners. The task items from the study were analyzed quantitatively to determine the frequency of correct/incorrect, misconception, misinterpreted and unattempted questions.

A follow-up interview as discussed was conducted between the researchers and each learner in an attempt to get an insight into the misconceptions that underlie identified errors. The researchers noted down all the explanations given by learners on how they solved the quadratic equations. The researchers audiotaped learners as per agreement. The researchers transcribed the interview the day after it was held for easy recalling of what the respondent said in the interview. The audiotapes and verbatim transcripts were then analyzed in an attempt to answer the second part of the research question, that is, what are possible reasons why learners make errors in solving quadratic equations?

Ethics and Rigor

All tasks done by the students were closely related to the quadratic equations section in the school syllabus. The interview questions were also closely related to this school task as well as the research questions.

To have access to the research site, the researchers wrote request letters to the Department of Education, School Governing Body and the principal to seek permission from them to involve their learners in the study. Since the learners were minors with an average age of 17 years, the consent of their parents was also obtained. The learners were also given letters to participate in the research. They were given consent forms to sign for the interviews and audiotaping. Permission was granted by the Department of Education and the principal to conduct the research in the school. According to Makonye (2011), results obtained without complying with proper ethical considerations are considered substandard. No learner was forced to participate in the research. Anonymity and confidentiality were insured by asking the participants to write fake names on their quadratic tasks.

RESULTS

Analysis was done to identify types of errors learners made when solving quadratic equations through the methods of factoring, quadratic formula, and completing the square. Errors learners made were identified and diagnosed. The errors identified were classified as procedural, conceptual, carelessness and interpretation errors. Error classification depended on how the learners make the error. The written task provided evidence of how learners think and reason when solving quadratic equations. The learners' solutions and explanations of their answers in both the task and the interview helped identify reasoning behind them (Osei 2006). The different types of errors were recognized from the quadratic task and learners' explanations during the interview.

Solution by Quadratic Formula

In Item 1.1, the learners were required to solve the quadratic equation: = 2(x +2), by use of a quadratic formula. The learners were supposed to expand the bracket and leave the equation in the standard form ax^2+bx+c, which is equal to -2x -4 = 0. After that, the learners were supposed to recall the quadratic formula and substitute a = 3, b = -2 and c = -4 in the formula. Simplification of the equation was to be done to come up with two real roots for *x* to two decimal places.

Below are explanations of some learners' responses in solving quadratic equations, which illustrated their errors and misconceptions in solving them by using a quadratic formula.

Learner 1: The learner stated the wrong values of b and c. The learner failed to recall the quadratic formula, and the numerator in that formula was not completely divided by 2a. In the last step, 6 in the denominator, which was cancelled with no reasons of doing that.

Learner 2: The learner failed to recall the quadratic formula and did not divide the whole expression of the numerator by 2a. The c value was wrong, substitution was incorrectly done and the learner included two equal signs in one equation. The learner ended up with+-14 after diving the -28 in the square root sign and a 6 in the denominator, which was wrong. The learner changed the +-14 to +14 and ended up x = 4.66 or x = -4.66.

Learner 3: The learner misinterpreted the equation by writing the equation: $3x^2=2(x-2)$ instead of $3x^2= 2(x+2)$. The learner failed to recall the quadratic formula. In step 2, the learner failed to respect the equality sign and put two equal signs in one equation. The values of a, b and c were wrong. The learner ended up with= and came up with two real roots for x.

Solution by Factorization

In Item 1.2 learners were instructed to solve the quadratic equation: (2x + 3)(3 –x) = 4, by factorization. Learners were supposed to use the FOIL method to simplify the brackets then collect like terms leaving the equation in the $ax^2+bx+c=0$ form. The factors of the first term: $2x^2$ are 2x and x, the factors of -5 the last term are 5 and -1. If they cross-multiply these factors, they give +5x and -2x, which will add up to -3x. After factorization the learners were supposed to equate the factors to zero and solve for x.

Below are three explanations of some learners' responses in solving quadratic equations, which illustrated their errors and misconceptions in solving them by factorization.

Learner 1: Here the learner simplified the brackets and ended up with $2x(-x) = 2x^2$, which is wrong. In step 3, -3x and 9 were changed into +3x and -9 unnecessarily. The learner ignored the equality sign and ended up with an expression instead of an equation. The learner simplified 6x +3x as 3x instead of 9x and also -9-4 is -13 not -5. The learner calculated $2x^2+3x$ and equated it to $5x^2$ but they are not like terms.

Finally, the learner divided by $5x^2 - 5$ to come up with x^2 leaving the -1 and failed to factorize the expression. No x values, the roots of the equation were not found.

Learner 2: Here the learner did not master the concept of an additive inverse, the number 4 was supposed to be subtracted on both sides. The learner tried to collect like terms without expanding of brackets. In step 2 and 3, the learner ignored the equality sign and ended up with an expression. The learner wrote $2x+x=2x^2$, which is incorrect. The learner reintroduced the equality sign but equating the LHS to x, which was introduced from nowhere on the right-hand side. The learner ended up with $2x^2+4=x$ and divided this answer by two to come up with x+2 =x, ended up with 3x=x, which is incorrect because unlike terms cannot be added.

Learner 3: Here the learner expanded the bracket correctly but did not equate it to zero in step 1 and introduced two equal signs in one equation. In step 3 the learner changed $-2x^2$ to $2x^2$ without dividing the whole equation by -1. The learner ended up with a wrong equation $2x^2 + 3x + 5 = 0$. The learner wrote $2(x + 5)(x – 2) = 0$ but 2 is not a common factor, the factorization was wrong hence the answers are incorrect.

Solution by Completing the Square

In item 1.3, the learners were requested to solve the quadratic equation $2x^2 –7x + 6 = 0$ by completing the square. The learners were supposed to first divide everything by 2 making the coefficient of x^2 equal to 1. In completing the square learners should ensure that the coefficient of x^2 is 1 and if it is greater or less than 1, they should divide by that coefficient before they could find the additive inverse of c both sides. Learners can then add half the coefficient of x both sides before the equation could be factorized the equation can then be factorized on the left-hand side and be simplified on the right-hand side. After that, the learners were supposed to transfer 3 to the left-hand side and make it -3. The learners were supposed to add half the coefficient of x squared to both sides and then factorize the expression. A square root was to be found for the term on the right and left-hand side. After that, the solutions of x were to be given.

Below are explanations of the learners' responses in solving quadratic equations, which illustrated their errors and misconceptions in solving them by completing the square.

Learner 1: Here, the learner transposed c as 3 instead of -3 and x was not written in step 2. She multiplied - and 3 by half the coefficient of x without squaring instead of adding. Later on she transposed the c back to the left-hand side and could not proceed from there, and hence no solutions for x were obtained.

Learner 2: Here the learner did not divide everything by 2, and half of 7 were added on both sides. The equation was simplified to 2x =+-2, 45 but how it was done cannot be followed.

Learner 3: Here the learner multiplied by half the coefficient of x without a negative sign. The learner did not add but multiply this half with the coefficient of x both sides. Because of this mistake, there were no solutions for x.

Quantitative Analysis of the Quadratic Tasks

Table 1 summarizes marks showing the number of learners who obtained marks from 0 to 5 in the quadratic tasks.

Table 1: Summary of marks for the number of learners who obtained marks from 0 to 5 in the quadratic task

Question	*5*	*4*	*3*	*2*	*1*	*0*	*No attempt*
1.1	5	8	5	3	4	7	0
1.2	8	5	2	3	7	7	0
1.3	6	1	1	4	6	10	14
Total	19	14	8	10	17	24	

All the learners attempted item 1.1 and item 1.2, which requires the use of quadratic formula and factorization. Using quadratic formula, fifty-six percent of the learners obtained a mark greater than fifty percent, in factorization forty-seven percent of the learners got a mark greater than fifty percent and in completing the square only twenty-five percent of the learners obtained a mark of fifty percent and above. This revealed that most of the learners are able to solve quadratic equations using a quadratic formula with some few difficulties encountered.

The errors learners displayed were categorized as conceptual, procedural, interpretation and carelessness

In each item, the highest numbers of errors were conceptual errors (see Table 2). This indicates that most of the learners fail to understand the concepts involved in the quadratic task. They fail to connect the relationships between concepts. Learners displayed a lot of procedural

Table 2: Frequency of each type of error per item

Item	*Conceptual*	*Procedural*	*Carelessness*
1.1	28	18	6
1.2	33	9	6
1.3	34	21	9
Total	95	48	21

Tasks

- Solve the following quadratic equations by using a quadratic formula
  $3x^2 = 2(x+2)$
- Solve the following quadratic equations by factorization
  $(2x+3)(3-x) = 4$

Solve the following

errors in item 1.3, followed by item 1.2. Here learners failed to carry out some calculations even though they understand the given concepts. Few careless errors were displayed in all items, and here it might be that the learners were careful in their workings.

Generally, the learners failed to recall the quadratic equations and could not substitute correctly in the formula. They put two equal signs in one equation indicating the beginning of their workings, and they also changed an equation into an expression. Learners failed to expand brackets correctly using the FOIL method. Learners conjoin unlike terms and struggle with addition, subtraction and multiplication of integers and change signs of terms unnecessarily. The additive inverse concept is not understood where learners need to balance an equation by adding or subtracting the same term on both sides of the equation. Learners cannot find factors of terms and complete the square.

Interviews with Learners

Quadratic Formula

Researcher: *Is this the quadratic formula*: *I can see you wrote* $(-2)^2=-4$. *Can you explain?*

Learner 1: *Yes, it's a quadratic formula. Ma'am,* $-2^2=-4$ *from the calculator you can even check.*

Researcher: *I can see in this step you wrote 4-48 = 44 not -44, why?*

Learner 2: *Because I was seeing a negative in the square root sign, so I thought of changing it to+44.*

Researcher: *Why are you putting two equal signs in one equation? Your division is it affecting only this part or was supposed to go a long way up to +-2?*

Learner 3: *Oh! It's a mistake, I am sorry. It was supposed to go a long way. Its like it shows that it is the same thing it's one thing.*

Factorization

Researcher: *I can see here that you wrote* $2x(-x)=2x^2$. *Can you explain this one, what was the reason of doing that?*

Learner 4: *I took it from the method of factorization, meaning that +×+=+, -×-=-, +×-=+ and -×+=+, that's why I ended with a positive by using those method of factorization.*

Researcher: *I can see here you have written* $2x^2+3x-5= 5x^2 -5$. *Can explain what was going on here?*

Learner 4: *That's what I did ma'am, I added the like terms and ended up with* $5x^2-5$ *and divided by the common factor 5, which lead me to my answer of* x^2.

Researcher*: In the statement there is =4 but in your statement there is no=4, what happened?*

Learner 3: *Ok like ma'am, I added everything up, I took the 4 and putted it within the brackets, I made like a one sum.*

Researcher: *If we go on to stage number 3...there was a 2x and x now its* $2x^2$, *can u explain?*

Learner 3: *I added this 2x to x and gave me* $2x^2$, *I just remove the x and brought it here,this x stands for the anonymous answer.*

Researcher: *You are now saying that x+2=x is equal to 3x, where did you get the 3x from?*

Learner 3: *I took the 2, divided it with* $2x^2$, *then I got x and 4 divided by 2 I got 2 then adding it up I got 3x so x = 3x.*

Researcher: *Now I can see there is a 2 outside the two brackets like,* $2x^2+3x+5 =2(x+5)(x-2)$ *what's going on here? Is 2 a common factor?*

Learner 5: *Not necessarily a common factor but the coefficient of x.*

Researcher: *So when you are factorizing you look at the coefficient of* x^2 *and factor it?*

Learner 5*: Yes ma'am.*

Researcher: *You simplified to come up with* $2x^2-3x-5=0$, *but at the end you wrote no solution, why?*

Learner 6: *Because factors of 5, its 1 and 5 only so when I add them I don't get a 3x.*

Completing the Square

Researcher: *Why did u divide by 2?*

Learner: *It's a rule ma'am. It's completing the square. It is always to have a half. I transposed the 3 to the left and put a half there and multiply the fraction.*

Researcher: *Explain what's going on in this one* $2x^2-7x+(\times 7)=-6+(\times 7)$?

Learner 5: *Ma'am, it's a mathematical rule to multiply the second value by half, which therefore when you do this on this side of the equation you have to do that on the other side of it as well.*

The above transcripts show that some learners confuse the quadratic formula, while some struggle with signs when they are adding, subtracting and multiplying. The additive inverse is not known in some instances. Division of terms is a problem. Finding factors of expressions is an obstacle to learners, as some know but mix up the signs in between the terms. They did not grasp well the concepts of completing the square. The learners were able to state their reasons why they make errors. The reasons are forgetful, not understanding and mixing up concepts. Some learners make mistakes because of lack of conceptual and procedural knowledge.

DISCUSSION

From the results, certain themes emerge. The researchers discuss them in the following section.

Theme 1: Misconception with Equality

Some learners ignored the equality sign and change the equation to an expression. Some do that because they were confused by the word factorize, some do that carelessly, some do that because they do not know the correct procedure. As a result of this, learners did not come up with the roots of equations. Essien and Setati (2006) believe that the limited comprehension of what the equal sign means is one of the challenges students face in the understanding of algebra.

Theme 2: Misconception with the Distribution Law

Learners fail to apply the distributive laws correctly. Learners did not multiply terms correctly. Some learners mixed up signs of terms. They did not know the correct sign to use. Some put a positive when it's negative or vice versa. Some learners were totally lost on the concept of the distributive laws. They did not multiply brackets but simply put terms together. For instance, for (2x-3)(3-x) = 4, some learners wrote the answer as (2x-3+3-x+4), which is no longer an equation but an expression. Learners have problems in the removing of parenthesis.

Theme 3: Misconception with Like and Unlike Terms

Learners fail to understand the concepts of algebraic terms, and do not understand the concepts of like and unlike terms. They conjoined unlike terms. In algebra, 2+x is not equal to 2x (Booth, 1986). Learners tend to close algebraic expressions by combining unlike terms. For instance, some learners wrote $2x^2+ 3x=5x^2$. This showed that the learners did not master the concepts of algebraic expressions from grade 9 and 10. In the conjoining process, the learner tended to take the highest exponents in those terms. They took x^2 instead of x for the coefficient of 5.

Theme 4: Misconception of Addition, Subtraction and Multiplication of Integers

Learners fail to add, subtract and multiply integers. They do not know the concept that, x-=+, -x+=-, +x-=- and +x+=+. There was a mix up of signs. Some learners just change signs of terms unnecessarily. Learners do not know the sign to use especially if the terms have different signs. For instance, for -9+5, learners wrote the answer 4 or -4, 4 is correct but some gave that answer by just guessing. Some did not substitute the correct b=-2 because they thought the negative is already there.

Theme 5: Misconception in Factorization and Recall of the Quadratic Formula

Learners fail to recall the quadratic formula it might be due to memorization of the rule. Also, some learners fail to factorize concepts, and factors concepts in grade 9 and 10 were not mastered.

Theme 6: Misconception in Substitution

Learners fail to state the correct values of *a*, *b* and *c*, and hence the substitution was incorrect. Some stated the values correctly but failed to simplify the terms since they struggle with integers. Some know the values to substitute but mix up the terms. For instance, learners exchanged values like *a* used as *b* and vice versa.

Theme 7: Misconception with Additive Inverse

Learners fail to change sign of terms if they add or subtract a term from an equation when eliminating terms. The term to be eliminated is positive, and the additive inverse must be negative and vice versa.

From analysis of the research data for this study, it appears that participants had a problem in solving quadratic equations. The trend in the learners' performance revealed that most learners have difficulties to solve the quadratic equation through completing the square. Solving the equation by factorization was less challenging for learners. This was evidenced by the number of errors produced in these items, which were 64 and 48, respectively. Fifty-one percent of learners recalled the quadratic formula but failed to substitute the correct values of a, b and c. Sixty-eight percent of learners had not mastered the substitution procedure. Learners have problems in removing parenthesis, and forty-three percent had mistaken in recalling the formula.

CONCLUSION

The researchers now look back on the research questions in order to answer them.

What is the Nature of Errors made by Learners in Solving the Quadratic Equation?

The researchers identified different types of errors made by learners, namely, procedural, conceptual, interpretation and carelessness errors. These were displayed by learners in their workings when solving the quadratic tasks. Most learners failed to understand concepts involved in solving quadratic equations. They do not understand the concept of equality, additive inverses, square roots, algebraic expressions, distributive laws and division, multiplication, addition of integers. The learners failed to connect the relationships between concepts like the concepts of like terms and expansion of brackets using the FOIL method when solving the equations. This resulted in the errors identified as conceptual errors.

Learners recalled the quadratic formula, they understood mathematical concepts involved in the task like factorization, completing the square but these learners failed to carry out some appropriate procedures or calculations. They failed to substitute in the quadratic formula, they failed to collect like terms, they failed to add integers properly, they failed to complete the square because of lack of procedural knowledge, they introduced two equal signs in one equation, and they changed an equation into an expression. This resulted in the errors called procedural errors. Some learners knew the mathematical concepts and procedures to apply on the quadratic task. However these learners failed to solve the quadratic equations because they were careless. For instance, learners knew that -(-2) = 2, but unknowingly they wrote -2. The researchers realized that learners were careless in solving the equations from the explanations they gave during the interview. Most of them said they knew what to do in the task but they were careless in their workings. They wrote correct statements in their workings in certain steps, which they changed unnecessarily to something wrong. This resulted in the errors named carelessness errors. Another type of error identified was interpretation errors. This resulted from learners misinterpreting the quadratic task. Some learners saw the word factorization and changed the equation into an expression. Some learners just introduced another *x* thinking that it was the one to be solved.

What are the Possible Reasons for Learners to make those Errors?

In the study, the learner-educator interviews revealed some reasons why learners make errors in solving quadratic equations. The reasons include:

- Language difficulties
- Lack of mastery of directed numbers
- Wrong generalizations and rigidity in thinking
- Use of wrong rules or strategies

These factors result in learners not being mathematically skillful to solve the quadratic equations.

Generally learners have difficulties in solving quadratic equations. This is supported by the amount of different types of errors displayed on their scripts. Also, the learners said that they have problems with quadratic equations. The revelation was obtained from the interview and some of the reasons for failing to solve the equations are shown above.

RECOMMENDATIONS

The major findings in this particular study revealed that the participant South African learners face some difficulties when solving quadratic equations. It is probable that other mathematics learners will experience the same problem since they are exposed to the same curriculum.

The researchers recommend that more research be undertaken to determine the impact of teaching when learners' identified errors and misconceptions in solving quadratic equations shown in this study are used as instructional resources.

LIMITATIONS OF THE STUDY

The participants included 32 learners from one class in one school. The findings may not be a true representation of the problem being explored. Findings of a single case study may fall short in their representativeness. Instead of using one class from one school, the researchers could have included more participants of different schools. The interview sample consisted of six learners, and the interviews were not very broad to cover all the possible reasons for learners' errors and misconceptions in quadratic equations. Also, the researchers' questions were very suggestive and such learners were directed to certain answers. Some learners' statements in the interview were vague.

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