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# Improving Proficiency in Mathematics through Website-based Tasks: A Case of Basic Algebra

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**KEYWORDS** Five Strands of Proficiency. Basic Algebra. Scaffolding. Mental Conflicts. Website-based Tasks

**ABSTRACT** Understanding how university students make sense of mathematics is always of concern to their lecturers. A group of lecturers at a South African university studied written responses of first year students to explore what mental conflicts arose during their students' formulations of solutions to assigned tasks. The mental conflicts as illustrated by their responses were categorized. One of the sections considered was basic algebra which is the case discussed in this paper. After detecting the common conflicts, the lecturers designed website-based tasks. The tasks were meant to target mental conflicts with the hope of removing these from students' cognition. The website-based tasks were designed in accordance with the principle of scaffolding. The researchers analysed the various conflicts within the framework of Kilpatrick's five strands of mathematical proficiency. In this paper the researchers discuss the design of the activities and interactive collaborations during tutorial sessions with students. We found that the website material seemed to have helped in removing common mathematical conflicts of some of our undergraduate mathematics students. Also it was found that non-mathematical terminology used could contribute to students' mental conflicts.

## INTRODUCTION

South African learners from Grades 10 to 12 take either Mathematics or Mathematical Literacy. These are two separate learning areas from the exit band of the South African secondary education system. Learners from Grade 10 onwards are supposed to take either but not both. The School of Mathematics, Statistics and Computer Science at the University of KwaZulu-Natal (UKZN) accepts learners who passed Mathematics, not Mathematical Literacy. Mathematical Literacy was conceived by the South African Department of Education (2003a:10) as

*… drive by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems.*

Mathematics was defined within the South African National Curriculum Statement (Department of Education 2003b: 9) of the Further Education and Training phases as:

*Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. It is a distinctly human activity practiced by all cultures. Knowledge in the Mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change.*

This definition has many features which coincide with the five strands of mathematical proficiency informed by the work of Kilpatrick et al. (2001). In our detection of mental conflicts (mathematical errors or misconceptions) we associate them to these strands. Some studies related to exploring how university students develop mental constructions of concepts in real analysis (Brijlall and Maharaj 2009a), concepts of continuity of single valued functions (Maharajh et al. 2008; Brijlall and Maharaj 2009b) and the concept of a limit (Maharaj 2010) were carried out at the UKZN. Another South African study concerned with university students' learning of continuity and limits was carried by Bezuidenhout (2010). Undergraduate students' errors in calculus were studied in a South African context by Bowie (2000). More recently with the introduction of Outcomes Based Education (OBE) there was a concern relating to the extent of learner preparedness, to study first year university mathematics. Engelbrecht et al. (2010) studied the preparedness of first year engineering students at a South African University. They found that students who were exposed to the OBE curriculum: (1) were weaker than their predecessors with regard to mathematical and content related attributes, but (2) there were positive indications that those students adapted and developed over a semester. A report on the national senior certificate mathematics examinations in South Africa indicated that "a considerable number of centres are still producing candidates who do not understand basic subject content" (Department of Basic Education 2011: 98). The paper by Maharaj and Wagh (2014) focused on the need for pre-calculus mathematics'diagnostics and remediation for first year university students. They suggested that for students who lacked the necessary skills and knowledge, diagnostics and remediation could be provided in an online format. In a case study involving government universities in the Kingdom of Saudi Arabia (KSA), Joshi (2014) found that the computer played a pivotal role in improving the quality of education. The finding of that study was that ICT and e-learning offered opportunity to raise educational levels. That finding of the paper by Joshi (2014) was also a factor which motivated this paper. Within this context we envisaged that web-based support materials would help lead to a smoother transition from secondary school to first year university mathematics.

## Background

During July and August 2011, lecturers and tutors for the first year mathematics modules offered by the School of Mathematics Sciences at the UKZN were asked to provide feedback on possible reasons for poor student performance. Feedback received indicated that many students lacked basic skills and knowledge that they should have developed during their schooling. Further, from informal discussions with lecturers it seems that the quality of students coming from our schools is getting worse. The lack of basic skills and knowledge referred to, are prerequisites for the study of the first year university mathematics modules. These basic skills and knowledge were detected to belong to the following sections: (1) algebra, (2) functions, and (3) reasoning using symbols and connectives.

With this in mind the Maths e-learning research group for basic algebraic skillsat UKZN formulated the following research question: *How should suitable web-based support material be developed and implemented to assist in removing common mathematical conflicts of undergraduate mathematics students?* It was in this context that the support material wasdeveloped and uploaded for the first semester beginning in January 2012. The development of the support material focused on: (1) relevant notes based on the common mathematics conflicts identified, (2) online self-practice exercise and tests, (3) links to relevant online sites, and (4) further homework exercises. Students who studied the module Mathematics for the Natural Sciences were given access to the module website, which used a Moodle platform. The module homepage is shown in Snapshot 1. That homepage contained the sections: (1) Notices, (2) Information and Handouts, (3) Basic Knowledge and Skills for Mathematics, (4) Functions and Calculus, (5) Basics and Matrices, (6) Tutorials, and (7) Test and Exam Papers.

The plan was to get students started on the notes and practice exercises as well as tests as soon as possible. This was so that they could do something about their lack of basic knowledge and skills. Also to ensure that the required basic knowledge and skills to study university mathematics were in place. In Figure 1, under the section on Basic Knowledge and Skills for Mathematics, students could click on the given link which took them to the webpage displayed in Figure 2. There they could access the basic knowledge and skills for (1) Basic Algebra, and (2) Functions. In this paper we focus on the design and implementation of the former.

## Relevance of study to South African Educators

In South Africa many of the students attending universities have English as their second language and come from very disadvantaged backgrounds. In a studyconducted at the University of Pretoria, researchers concluded that there was no significant difference in performance betweena group of first language mathematics students and second language mathematics learners (Gerber et al. 2005). At UKZN English is the medium of instruction and stu-

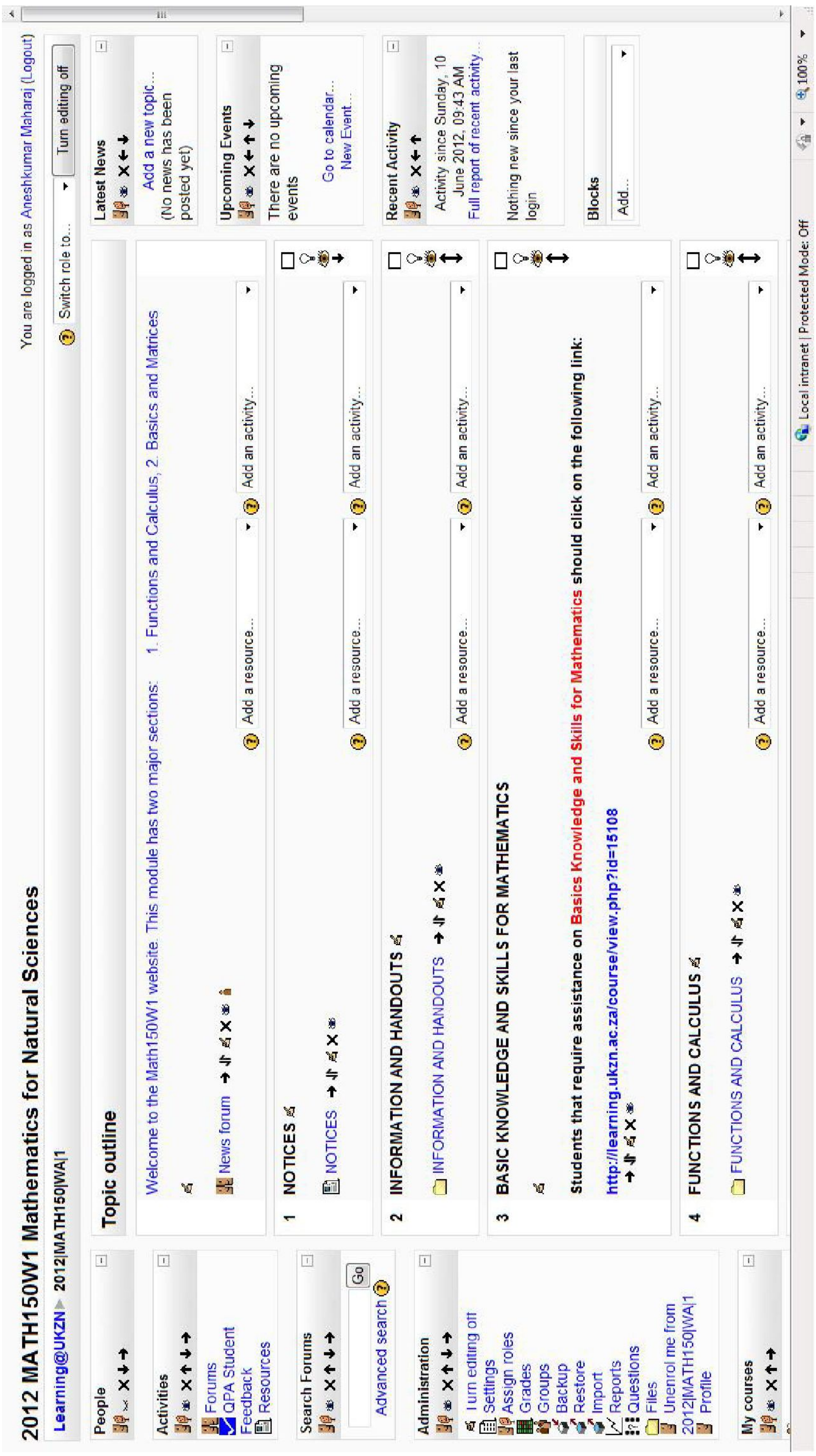


**Fig. 1.** Homepage for mathematics for natural sciences

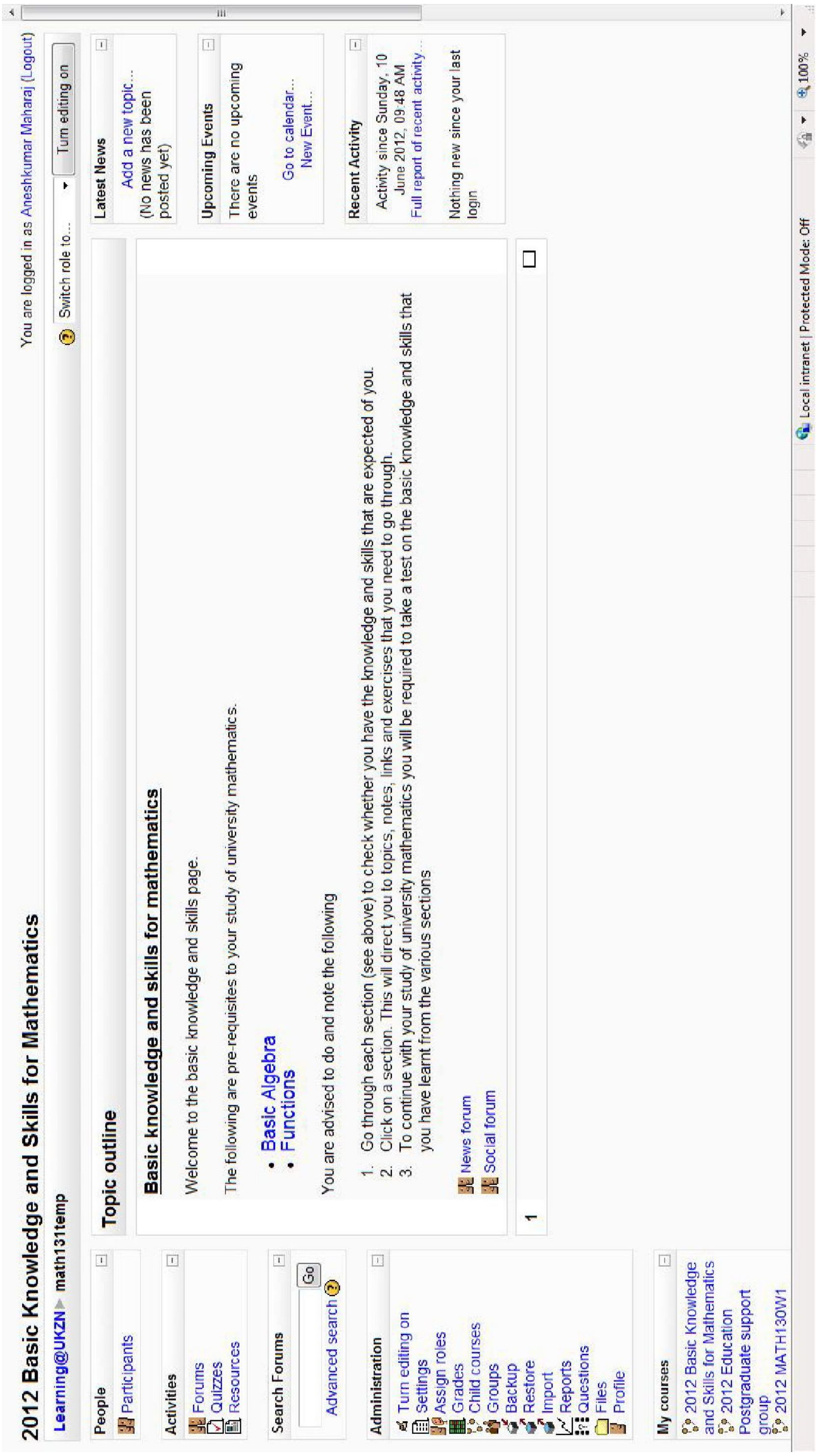



**Fig. 2. Webpage for basic knowledge and skills for mathematics**

dents come from different schooling backgrounds. The researchers believe that the language of instructionand the students' poor socio-economic background result in first year mathematics students performing poorly. Also, the researchers believe that this leads to arelatively high drop-out rate. The researchers felt therefore that the development of additional resource materials was vital to reinforce the students' school mathematical knowledge as well as improve the pass rate at the first year levelespecially. Furthermore, the researchers hope is that this will enhance students' understanding of the subject in subsequent levels at university.

## Theoretical Framework

Kilpatrick et al. (2001) describe five strands of mathematical proficiency, namely: Conceptual understanding; Procedural Fluency; Strategic Competence; Adaptive Reasoning and Productive Disposition. In this paper the researchers provide examples of the observed algebraic errors in each strand. Specially the researchers detected and analysed common mathematical conflicts, within written responses of first year engineering students. Most of the observed algebraic conflicts were in Kilpatrick et al.'s (2001) *procedural fluency strand*. Examples of some of these are discussed in this paper. These errors are named here to avoid repetition later.

*Conceptual Understanding:* this refers to comprehension of mathematical concepts, operations, and relations. Kilpatrick et al. (2001) argue that a significant mathematical indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. Van de Walle (2007) defines *understanding* as measure of quality and quantity of connections that an idea has with existing idea. Symbols, charts, graphs and diagrams are powerful methods of expressing mathematical ideas and relationships. Conceptual knowledge is knowledge that consists of rich relationships or webs of ideas (Hiebert and Carpenter 1992; Hiebertet al. 1996).

An example of an error of this strand is shown in Error Type 1. It seems here that the student thinks that when we subtract fractions we simply subtract the denominators to give the answer. His/her incorrect comprehension of the operation of subtraction of fractions (with different denominators) hampers his/her correctness in obtaining the correct solution. The student does not realize that to subtract two fractions with different denominators, a common denominator is required.

*Error Type 1:* Error involving algebraic processes with fractions

$$\frac{1}{a} \quad \frac{1}{b} = \frac{1}{a-b}$$

*Procedural Fluency:* This refers to knowledge of procedure, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently. Van de Walle (2007: 28) defines procedural knowledge of mathematics as "knowledge of rules and procedures used in carrying out routine mathematical tasks and also the symbolism used to represent mathematics". Skemp (1978) refers to this as instrumental understanding of ideas that are isolated and essentially without meaning. We found that most of the observed errors fell in this strand. In fact the error involving algebraic processes with fractions also overlap with this strand. Another error of this strand is indicated as Error Type 2. Many students had made this error. They 'cancelled' the "*h*" with the first term of the numerator only.

*Error Type 2:* Error involving algebraic processes in factorisingwhen *h* is not equal to 1

$$\frac{ah + b}{h} = a + b$$

What students committing this type of error fail to realise is that divisibility by "h" would only be possible if the numerator was for insistence (ah + bh). The students do not realise that to divide a common factor with the denominator, the numerator needs to have a common factor in both the terms of the numerator (as a consequence of the distributive property). Another common error that belongs to this strand is indicated in Error Type 3.

*Error Type 3:* Error involving algebraic processes with exponents

$$x^3 \times x^{-1} = x^{-3}$$
$$x^3 \times x^3 = x^9$$

In both examples in Error Type 3, the students' thinking was consistent. In each case, it appears that they thought if the bases are the same then they can multiply the indices to obtain the answer.

*Strategic Competence:* Is about following a number of steps leading to the solution of a problem. Specifically, this entails the ability to formulate Mathematical problems, represent these mathematically, and thereafter solve them. This strand is similar to what is referred to as problem formulationand problem solving. This means that students need to initially establish what the problem is and use mathematics to solve it.

*Adaptive Reasoning:* This refers to the capacity to think logically about relationships between concepts and situations. It includes knowledge of how to justify a conclusion. In the task that was given to learners in this study, learners were required to justify their mathematical claims and explain ideas in order to make their reasoning clear in their response and to the interviewer, during the interview. The concepts $\sqrt{x^2-9}$ and $\pm(x-3)$ in the first example in Error Type 4 are not related and generally no justification is made by the students in relating these two entities. Here the student believes that $x^2-9=(x-3)^2$ and consequently simplifies by taking the square root on both sides of the equality.

*Error Type 4:* Error involving algebraic processes with surds

Example 1: $\sqrt{x^2-9}=\pm(x-3)$

Example 2: $\sqrt{x^2+9}=x+3$

In example two of Error Type 4, the student probably believed that $x^2+9=(x+3)^2$ and consequently simplified by taking the square root on both sides of the equality. This type of error also occurs when the students are confronted with inequalities as shown in Error Type 5.

*Error Type 5:* Error involving algebraic processes with inequalities

Example1: $x^2 \geq 9 \Rightarrow x \geq \pm 3$

Example 2: $abc \leq 0 \Rightarrow a \leq 0$ or $b \leq 0$ or $c \leq 0$

In example 1 of Error Type 5, the student has taken the square roots on both sides of the inequalities to obtain the result. In example two of this Error Type, the student has made this mistake by thinking that the inequality in the expression is similar to the equality in the equation *abc =0.*

*Productive Disposition*: This is a strand which includes all the other strands. For example, as students build strategic competence in solving non-routine problems, their attitudes and beliefs about themselves as mathematics' learners become positive. The more mathematical concepts they understand the more mathematics makes sense to them (Kilpatrick et al. 2001). So, in essence productive disposition is reached when students see sense in mathematics; to perceive it as both useful and worthwhile; to believe that putting an effort in learning mathematics pays off; as well as see themselves as an effective 'learners and doers' of mathematics.

Figure 3 gives our overall strategy to address the Error Types discussed. At the beginning of the semester, twelve out of fifty-two lectures were scheduled to cover the algebra basic skills and knowledgefollowed by a section dealing with functions. Five of those twelve lectures focused on the algebra skills and knowledge, which is the focus of this paper. During those lectures students were exposed to activities and classroom discussions on examples relevant to basic algebra. A PC tablet was used during lectures to develop on electronic lecture outlines that were available to students, before the relevant lectures. An electronic outline indicated the concept(s), relevant activities and examples for discussion. During the lecture the tablet was used to expand on the relevant outline and to record important points that emerged from activities and classroom discussion. For the tutorial problems exercises similar to those discussed in class were set. PDF copies of the completed lectures and relevant homework problems were available to students on the module website. Students were informed that they had to engage with the exercises and come to the tutorials with their attempts. Before attending their formal group tutorials students could consult with support tutors, so that they were adequately prepared for the formal tutorials. Figure 3 (adapted from Maharaj 2012) indicates that online material, the module website and the use of technology were used to address the lack of preparedness of students to study calculus. Built-in into the design and implementation of tutorials was that students should get the most out of the tutorials, since the researchers saw this as a key to student success. It was planned thatthe design of the tutorials and the tutoring skills of our tutors; during formal and informal tutorial settings; were to facilitate the development of student skills, learning techniques, participation and success. These imply that overall strategy for all of this to be possible was driven largely by online material on the module website and the use of technology.

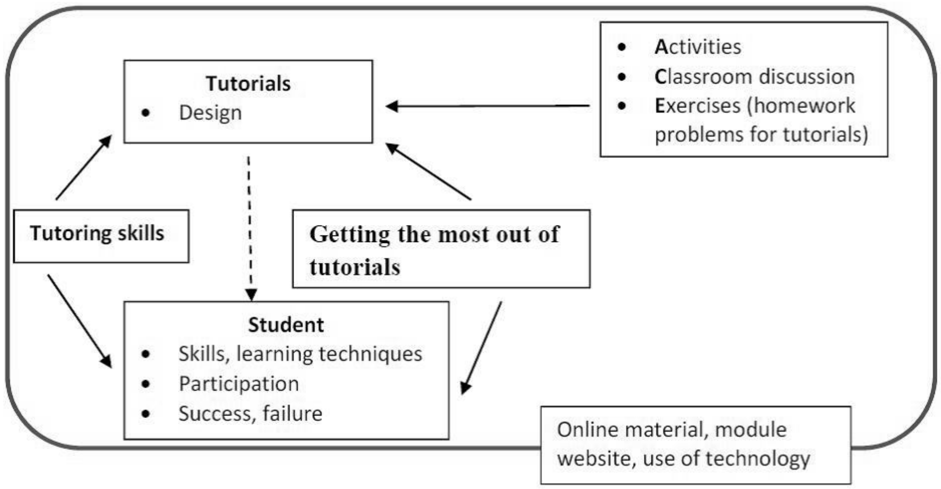



**Fig. 3. Overall strategy to address the lack of preparedness of students**

## METHODOLOGY

This section focuses on: (1) Research design, (2) Design of online learning materials after detection of common errors,(3) Informal Interviews and classroom observations, and (4) Participants, articulation of questionnaire and formal interviews.

### Research Design

A larger research project targeted the development of Basic Knowledge and Skills online website materials required for the study of first year university mathematics students, at the University of KwaZulu-Natal. This paper reports on the development of the material for algebra, and in particular the use there of by students studying the module Mathematics for the Natural Sciences. However, the study could not be completed just with them alone. According to Cohen et al. (2005), there is no single prescription for data collection instruments to be used. Using this principle as a guide, the study had divided the data capture in the learning phases interrogating (a) design of online learning materials after detection of common errors, (b) participant observations, (c) questionnaires and interviews, and (d) photographic clips.

### Design of Online Learning Materials after Detection of Common Errors

The design of activity sheets using scaffolding was used extensively in the study by Brijlall and Isaac (2011) and Brijlall (2011). In this study we have developed online multiple choice questions (OMCQ); see Figure 4; to assess whether or not the web-based support material has been successful in removing the students' mathematical conflicts. In designing the items we used similar strategies and adopted the principle of scaffolding derived from the work of Vykotsky (1978) and elaborated by Zhao and Orey (as cited in Lipscombe et al. 2008).

Figure 6 indicates the processes that led to the development of the online support material. A team of researchers of the Mathematics Education Research Group explored the relevant Basic Knowledge and Skills required for the study of calculus. The exploration included (a) informal conversions with lecturing staff and tutors on what was lacking, (b) emails to first semester lecturers requesting information on basic knowledge and skills that they found to be lacking during the marking of the June 2011 examinations, and (c) a study of relevant precalculus sections in our prescribed textbooks. Based on these the team decided to group the feedback to focus on basic algebra, functions, and logic and reasoning. It was decided to form three sub-teams to focus on each of these. The algebra team then used relevant feedback to determine the algebra knowledge and skills required to study first year university calculus. The relevant feedback together with a discussion of the common Error Types were used to inform the design of online material. The online

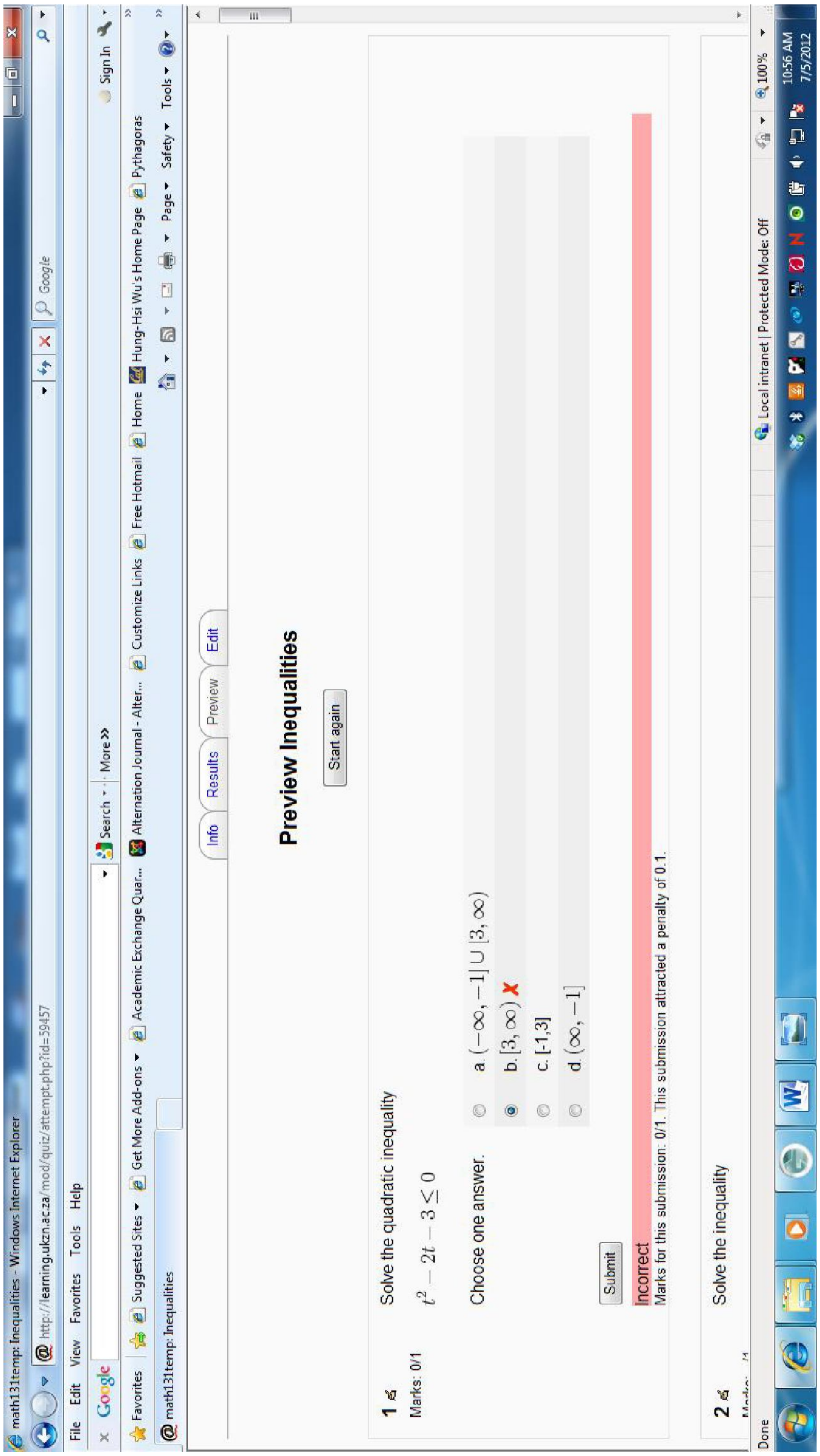



**Fig. 4. OMCQ for inequalities**

material includes notes, relevant links to other websites, exercises, answers and quizzes. Figure 5 provides an illustration of how the material was packaged for the students.

According to Zhao and Orey (as cited in Lipscombe et al. 2008) scaffolded instruction has six general elements namely: "sharing a specific goal, whole task approach, immediate availability of help, intention assisting, optimal level of help and conveying an expert model" (Lipscombe et al. 2008: 5).

(a) *Sharing a Specific Goal:* It is the responsibility of the more knowledgeable one (MKO) to establish a goal and share it with his students. For example it is the responsibility of the lecturer to refer a student to online material that is available to him or her (see Fig. 7a). Allowing for input from the students on the shared goal enhances their intrinsic motivation. It also assists the learner to master the goal that has been set, by developing new skills. In this way, learning is also enhanced and the student sees that he can attain success by putting in the required effort.

(b) *Whole Task Approach:* The focus of this approach is on the ultimate goal that is to be achieved. In this approach each task is seen as how it relates to the ultimate goal. This approach is only effective if the student does not experience any difficulty in any of the minor tasks in order to achieve the ultimate end result.

(c) *Immediate Availability of Help:* In scaffolded instruction success is important in order to control the frustration levels of the students. If the MKO provides assistance and support timeously, then the stu-


**Fig. 5. Webpage giving links to notes, links, exercises, answers and quiz for the topic inequalities**

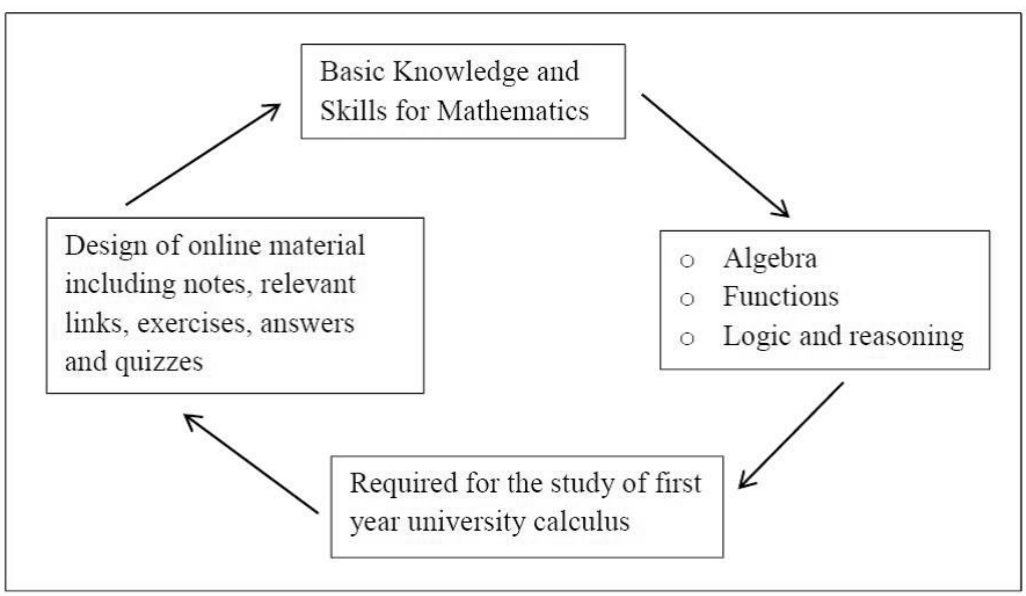



**Fig. 6. Processes that led to the design of the online support material**

dent experiences success and this motivates the student to be more productive.

(d) *Intention Assisting:* In the scaffolding process it is important to provide assistance to the learner's present focus in order to assist him with his current problems. In this way a more productive learning environment is developed as the learner pursues his current task. Sometimes, however, it is necessary to redirect the learners thought processes, if they do not coincide with the current task at hand. The MKO needs to be aware of the different methods of completing a task and if the learner's method allows him to complete the task in a different way, then the MKO must accept this method and allow the learner to proceed with the least amount of assistance. If however the learner needs constant assistance then the MKO could turn to coaching to assist the learner.

(e) *Optimal Level of Help:* The level of assistance that is provided to the learner must match what he is able to do. The learner must get just enough assistance in order "to overcome the current obstacle, but the level of assistance should not hinder the learner from contributing and participating in the learning process of that particular task" (Lipscombe et al. 2008:7). Photograph 2 in Figure 7 shows students interacting with the web-based support material and assisting each other.

(f) *Conveying an Expert Model*: An expert model provides the learner with an example of how to accomplish a particular task. The techniques that are to be used for completing the task are clearly expressed.

The design of the website-based material took much of the above into account.

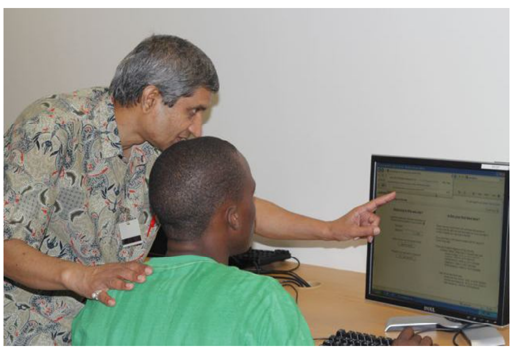


**Fig. 7. (i) Sharing ideas**

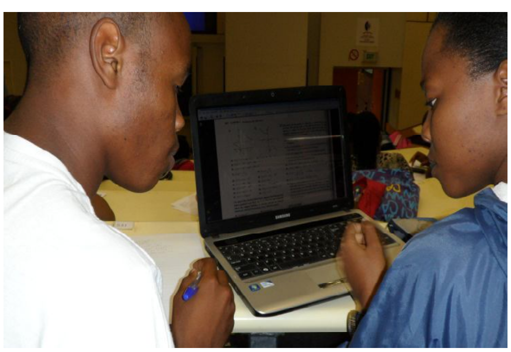


**Fig. 7. (ii) Sharing ideas**

## Informal Interviews and Classroom Observations

When attending the tutorial sessions for the Mathematics 150 module, the researchers recorded interesting pertinent observations. Theresearchers also spoke to some students on an informal level but recorded relevant outcomes of such interactions.

## Participants, Articulation of Questionnaire and Formal Interviews

A random sample of 51 first-year Math150W1 students was chosen to participate in this paper. These students were studying the module Mathematics for the Natural Sciences which introduced the fundamental principles, methods, procedures and techniques of mathematics. This module is a pre-requisite for students studying chemistry, physics, biology, zoology, optometry and pharmacy. The questionnaire, see Appendix,administered to the 51 students was designed to gather as much data as possible to answer the research question. The questionnaire was divided into two sections: (a) general questions, and (b) specific questions, related to basic algebra content. After going through the student responses to the latter section, the research-

**Table 1: Frequency of student visit to Math150 website (n = 51).**

| *Not at all* | *Up to 3 times a week* | *More than three times a week* |
|---|---|---|
| 0 | 35 | 16 |
| 0% | 68.6% | 31.4% |

ers interviewed 10 students to get further clarity on their responses.

## RESULTS AND DISCUSSION

This is presented under the following subheadings:(1) general questions on the use of website support material and (2) specific questions based on common errors made in basic algebra.

### General Questions on the Use of Website Support Material

Table 1 shows that all the participants visited the website at least three times a week. This suggests that the design and development of the website materialwas not in vain, since students made use of the website.

With regard to whether the material on the website was easy to interpret, the responses are summarized in Table 2. This table shows that (a) just less than half of the students found the material on the website easy to interpret, (b) about half of the participants found the material slightly difficult to interpret, and (b) one student found it difficult to interpret the material. That student wrote *"because there is no one to help me and its very hard to get to the answers"*. This seems to imply that this student did not consult with his group tutor nor the support tutors, see the write-up for Figure 3 which indicates the support structures that were available to students. Those support structures were provided keeping in mind the principle of scaffolding, dealing with *availability of help* (Lipscombe et al. 2008). With regard to those respondents that found the website material slightly difficult, one student wrote *"I don't have easy axcessto the internet on a daily basis because I live off campus."* This response does not really deal with difficulty on interpreting the material on the website, but rather accessibility to engaging with the material on the website. Another response in this category (slightly difficult) was *"sometimes the video is too fast or sometimes too slow, can't ask questions to the video"*. It seems that this student was unaware that the video can be paused, to possibly write notes, or parts of it that were not understood could be replayed. The part of the response dealing with *"can't ask questions to the video"* indicates the need for a human component, which was provided by the lecturer (during consultation times) and the tutors.

**Table 2: Responses on whether the learning material was easy to interpret (n = 51)**

| *Difficult* | *Slightly difficult* | *Easy* |
|---|---|---|
| 1 | 26 | 24 |
| 2% | 51% | 47% |

Some of the explanations provided by students who responded in the categories slightly difficult and easy were: (a) *The notes are detailed and easy to access in PDF formats.* (b) *Very convenient – easy to use.*(c) *Well laid out.* (d) *Things are in folders and easy to find and access.* (e) *It is displayed in an easy to understand step-by-step format.* (f) *Good breakdown of the material.* (g) *The notes are well organized according to lectures and is written in simple terms for everyone to understand.* (h) *Everything is set out well and what I need is easy to find.* These views in a sense indicate that the design of the website was reader-friendlyto them.Also note that the implication of their explanations is that an adequate scaffolding process was provided, since a more productive learning environment was developed for the studentsto pursues relevant tasks. This tied up with the principle of *attention assisting* advocated by Zhao and Orey (as cited in Lipscombe et al. 2008).

In response to the question on whether the support material for basic algebra was helpful to them, more than four-fifths of respondents indicated in the affirmative (see Table 3). Some of the explanations provided by such students were: (a) *It enabled me to solve any type of mathematical problem related to algebra easily.* (b) *It helped refresh my memory on the basics of algebra.* (c) *Gives step-by-step calculations which enables one to understand and follow.* (d) *I have improved on my class test – from test 1 to test 2.* (e) *I used them for practice.* (f) *It helps a lot because examples are clearly explained.* (g) *Because it explains the basic con-*

**Table 3: Responses on whether the material on basic algebra helped(n = 51)**

| *Support material helped* | *Support material did not help* |
|---|---|
| 44 | 7 |
| 86.3% | 13.7% |

*cepts of algebra in such a way that even people who did not do maths in high school would understand.* (h) *It was a recap on school work; making follow-up work easier to understand.* In particular the responses (g) and (h) indicate that the content of the support material provided sufficient assistance to overcome obstacles that they experienced. This addressed the principle of *optimal level of help* suggested by Zhao and Orey (as cited in Lipscombe et al. 2008).

Students who found that the content support material for basic algebra was unhelpful, less than one-fifth of the respondents (see Table 3), gave the following explanations: (a) *Not sure what or where that was, didn't view it.*(b) *Already understood it.*(c) *Because I already had these basics from high school and nothing was changed.*(d) *It would helped someone who was struggling with the concepts.* Their responses could be divided into two sub-categories. One pertains to unawareness of the web-based support material and the other relates to students who already knew the content, and thus concluded that it was unhelpful to them.

### Specific Questions Based on Common Errors Made in Basic Algebra

In the presentation of the data for the interviews we code the three researchers as A1, A2 and A3 in the order in which they appear on the first page of this article. The codes used for students who were interviewed in order to clarify what they wrote; were R1, R2, R3 and R4.

The item in Figure 8 focuses on an error involving algebraic processes with fractions, outlined in the theoretical framework (Error Type 1). Note that in this case student R3 was able to detect that (1) the student treated the operation of subtraction to apply only to the denominators, and (2) clearly states that the operation should apply to each of thefractions $\frac{1}{a}$ and $\frac{1}{b}$. This was further clarified in what transpired during the interview.

A3: *Could you tell me what was going on in the mind of the student?*

R3: *The student just subtracted denominators.*

A3: *Why did he do that?*

R3: *I think that he does know the rules of fractions.*

The reference by student R3 to "the rules of fractions" belongs to the strand *procedural fluency*, outlined in the theoretical framework (Kilpatrick et al. 2001). Her reference to "just subtracted denominators" displays her comprehension of mathematical operations in relation to entire fractions, which provides evidence of *conceptual understanding*.

The item in Figure 9 deals with an error involving algebraic processes in factorising, as outlined in the theoretical framework (Error Type 2). Student R2's written response shows he understands the notion of a common factor in an algebraic expression. Note that student R2 displayed *conceptual understanding* since he was able to identify the incorrect use of factorisation, in the context of the two terms relating to the numerator of the algebraic fraction $\frac{ah+b}{b}$. Observe that student R2 replaced the word *divide* by the word *cancel*. The following transpired during the interview.

A1: *What do you mean by 'cancel'?*

R2: *Divide*

A1: *Didn't the student divide?*

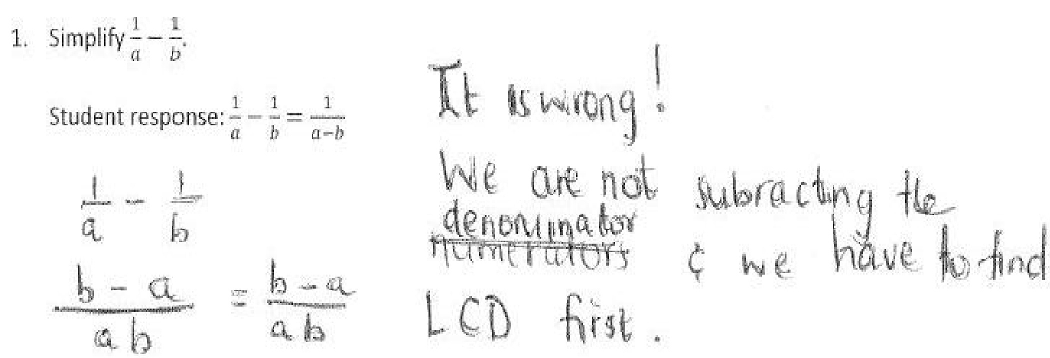



**Fig. 8. Comment of student R3 to item 1.**

Student response: Yes. $\frac{ah+b}{h} = a + b$

incorrect ~~Correct~~ as student took out 'h's as a common factor at the top whilst 'b' does not have 'h' as part of it ∴ 'h' cannot be taken out as a common factor to cancel ~~divide~~ with the bottom.

**Fig. 9. Comment of student R2 to item 2.**

R2: *I explained it that h is not a common factor in the numerator.* (pointing to his written response)

Those responses imply that student R2 used the words cancel and divide interchangeably, which is not good practice. In mathematics one of the valid operations is division; cancel is not an operation!

Figure 10 indicates the comment of student R1 to an incorrect student response which was based on the error involving algebraic processes with surds, as outlined in the theoretical framework (Error Type 4). Note that student R1 indicated that the "student is correct". With the intention of addressing the principle of *optimal level of help* suggested by Zhao and Orey (as cited in Lipscombe et al. 2008) the following transpired during the interview.

A1 writes $\sqrt{25-9} = \cdots$ R1 indicates verbally $\sqrt{25-9} = \sqrt{16} = \underline{4}$

A1 writes =.. $\sqrt{25\text{-}9} = (25)^{½} - (9)^{½}$ R1 responds verbally 5-3

R1: *Does not give you the same answer because there are terms* (pointing to the terms under the square root sign). *Not factors!*

This response indicates that student R1 realized the difference between terms and factors in the context of applying algebraic processes. The implication here is that to provide *optimal level of help* to such students, in the context of focusing their attention to the difference between factors and terms,some aspects of the online material developed need to be reconsidered. For example *pop-ups* could be used in an appropriate context to enquire if the student knows the difference between terms and factors.

The item in Figure 11focuses on an error involving algebraic processes with exponents, as outlined in the theoretical framework (Error Type 3). Note that in this case student R1 was able to detect that (1) the student response was incorrect and (2) was able to give an explanation leading to the correct answer. This seems to imply that the online support material developed and

Student response: $\sqrt{x^2 - 9} = \pm(x-3)$

$\sqrt{x^2-9}$

$= x^{2/2} - 9^{1/2}$

$= (x-3)$.

Yes Student is correct

**Fig. 10. Comment of student R1 to item 3.**

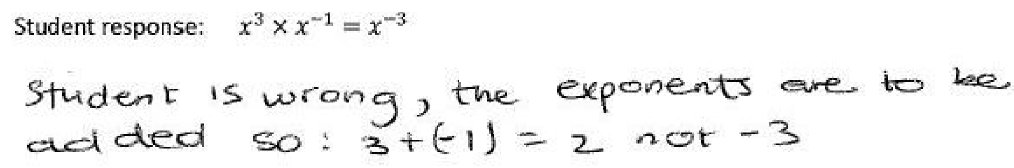



**Fig. 11. Comment of student R1 to item 4.**

implemented was of benefit to this student. That was supported by student R1's response to the question: *Does the module website help you?*

*It does. Even having the lecture outlines helps us to prepare and know what to expect during the lectures. …. We get the tutorials beforehand … and it helps … time management. This is the main factor for most of us. You think you have enough time but you don't.*

These two responses of student R1 imply that the online material developed and implemented addressed the principle of *whole task approach* (Lipscombe et al. 2008).

The item in Figure 12was designed to focus on a common error involving algebraic processes with inequalities, as outlined in the theoretical framework (Error Type 5). Note that the working of student R4 in Extract 5 implies that this student (1) knew that the student response to the item was incorrect, and (2) knew a correct procedure to solve the quadratic inequality, $x^2 \geq 9$. To further probe the former the following transpired during the interview.

A2: *You have the correct answer. Why did the student, in your opinion, write what he did?*

R4: *Student introduced square root to solve for x which was wrong because of the " $\geq$ " sign.*

A2: *What basically was he saying?*

R4: *The student treated the " " sign as an equal to sign.*

The last two responses of student R4 suggest that the student response *falsely generalized* properties associated with an equal to sign to be applicable to an inequality sign, a common misconception. In the design and implementation of the online support material for basic skills and knowledge for university mathematics, the basic algebra section focused on the following methods for solving inequalities (1) table method, (2) use of graph, and (3) use of number line[also referred to as sign diagram; used by student R4 in Figure 12].

The sign diagram method seems to be popular amongst many students, and reasons for this became apparent during the interview with student R1.

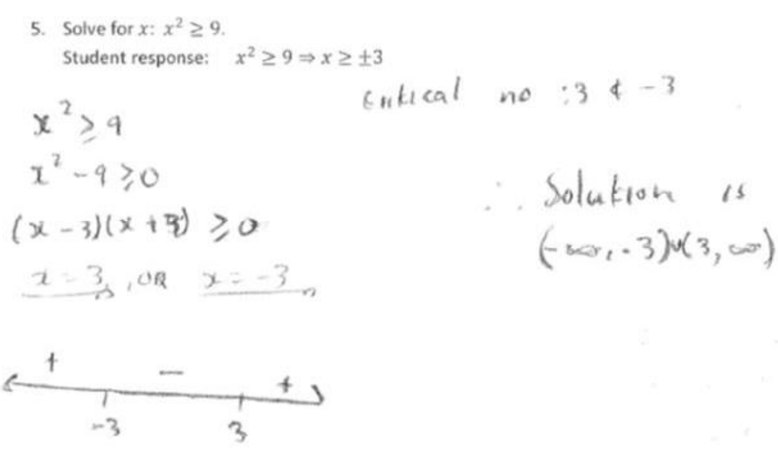


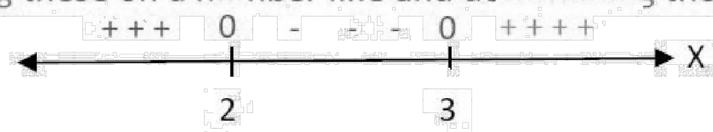

**Fig. 12. Comment of student R4 to item 5.**

Method 3[use of number line]

$$x^2 - 5x \leq -6 \Rightarrow x^2 - 5x + 6 \leq 0 \Rightarrow (x-2)(x-3) \leq 0$$

Critical value are: $x = 2 \ or \ x = 3$

Ordering these on a number line and determining the behavior of $(x-2)(x-3)$ we get:




So the solution of the inequality is $2 \leq x \leq 3$. In interval notation the solution is $[2, 3]$.

Explanation of how to determine the behavior of $(x-2)(x-3)$ on the number line above:
Consider $F(x) = (x-2)(x-3)$. $F(x) = 0$ at $x = 2 \ or \ x = 3$, insert 0 above these values
In each of the intervals formed we choose a convenient representative and use this to determine whether $F(x) > 0 \ or \ F(x) < 0$.
For $x < 2$ choosing $x = 1$ gives $F(1) > 0$ . So insert (+) in this interval.
For $2 < x < 3$ choosing $x = 2\frac{1}{2}$ gives $F(2\frac{1}{2}) < 0$. So insert (–) in this interval.
For $x > 3$ choosing $x = 4$ gives $F(4) > 0$. So insert (+) in this interval.

**Fig. 13. Discussion on the number line or sign diagram method for solving inequalities**

A1: *You spoke about the sign diagram. Where did you come across this?*

R1: *Lectures and notes on website. The graph or table way can also be used. But I find it easier to use the sign diagram. It really helps with a lot of other sections … finding where a function is increasing or decreasing … finding maximum or minimum value of a function …*

This indicates that the student was able to relate the use of a method discussed in the basic algebra online support material; see Figure 13; to other sections in calculus (for example, increasing or decreasing functions). It also became apparent that students were not confined only to the website support material that was developed. This is substantiated by the dialogue:

A1: *Where on the website did you find the sign diagram?*

R1: *Lecture notes* (referring to the notes on the website). *I also Google while working.*

Student R1 responded as follows when asked the question.

A1: *Is there any way the website can be improved?*

R1: *Put in more numeric examples for basics* (opens website for basic knowledge and skills for mathematics and indicates) *Here! Also more quizzes.*

Note that this student was able to detect her error (see discussion to Figure 10) when researcher A1 used numeric values instead of variables. The implication here is that future online support material should provide a smoother transition from arithmetic to algebra (generalized arithmetic). It also seems that students prefer more quizzes, possibly because of the almost immediate feedback; as indicated in Figure 5.

## CONCLUSION

The procedure we followed to design and implement the website material seems to have helped in removing common mathematical conflicts of some of our undergraduate mathematics students. In the year 2011 the web-based support materials were not available to students. The pass rate in that year was 74.9%. The pass rate in 2012 was 79.4%. Those students were exposed to the web-based support material. Most of the student participants (about 90%) indicated that the website support material was easy or slightly difficult to interpret. Reasons given for this included: (1) The notes were well organized according to lectures and was written in simple terms for everyone to understand. (2) Support material were in folders and easy to find and access. About 80% of the students indicated that the website material for basic algebra helped them. Here they pointed out that it: (1) helped improve test marks; (2) helped in the understanding of basic concepts of algebra; (3) served as an enabling factor in solving any type of mathematical problem related to algebra; and (4) served as a means to recap school work, making follow-up work easier to understand.

One of the unintended findings of this paper is that mathematical terms or concepts should not be loosely used, since this could lead to mental conflicts. For example, the words cancel and divide should not be used interchangeably when the operation in question is division.

## RECOMMENDATIONS

The conclusions indicate that the web-based support material benefitted the students. The researchers suggest such online support materials should be made available to first year university students who do not have the necessary basic knowledge and skills to study undergraduate mathematics. It is also recommended that similar online support material based on other topics studied at undergraduate level should be made available to students.

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## APPENDIX

## Questionnaire on use of Math150 website

***A. General questions***

*1. How often do you visit the website for Math 150?*

| Not at all | Up to 3 times a week | More than three times a week |
|---|---|---|
| | | |

*2. Is the material on the website easy to interpret?*

| Difficult | Slightly difficult | Easy |
|---|---|---|
| | | |

Explain your choice: ____________________

____________________

*3. Did the material on basic algebra help in your studies?*

| Yes | No |
|---|---|

Explain: ____________________

____________________

***B. Basic Algebra Questions***

In each of the questions below comment on the student responses provided:

1. Simplify .$\frac{1}{a} - \frac{1}{b}$

Student response: $\frac{1}{a} - \frac{1}{b} = \frac{1}{a-b}$

2. Can you simplify $\frac{ah+b}{1}$?

Student response: Yes. $\frac{ah+b}{h} = a+b$

3. Can you simplify $\sqrt{x^2 - 9}$?

Student response: $\sqrt{x^2 - 9} = \pm (x-3)$

4. Simplify . $x^3 \times x^{-1}$

Student response: $x^3 \times x^{-1} = x^{-3}$

5. Solve for : x: $x^2 \geq b$

Student response: $x^2 \geq 9 \times \geq 13$