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Error Discourse Analysis in Solving Quadratic Equations by Completing A Square and Learner Perceptions of Sources of Errors
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KEYWORDS

ABSTRACT This paper discusses the errors leaners displayed is solving quadratic equations by completing a square in grade 11 mathematics. The results revealed that learners in the schools participated in this study experienced problems in solving quadratic equations by completing a square. The study was conducted in (N = 5) South African schools of Limpopo Province in Capricorn district. The main aim was to diagnose errors learners made in solving quadratic equations by completing a square and found the reasons why those errors occurred. Learners' scripts (N = 65) were sampled and (N = 50) learners participated in focus groups in the five schools to diagnose those errors. The data collected from the learners' focus group interviews revealed that the teaching approaches used by their teachers contributed towards their errors they made in solving quadratic equations by completing a square. From the focus group interviews most of the learners claimed that they were not given an opportunity to discuss the concepts and some indicated that they did not have enough time to practice their work. Other reasons found in the five schools are – learners are afraid to be mocked at by their peers during the lessons, learners themselves not committed to their school work, some have parental role to play at home, and some teachers are not committed as they come to class late and not controlling their work. This study suggests that teachers have to know and understand learners' errors in solving quadratic equations by completing a square and mathematics hostically to know the type of instructions in teaching this concept.

1. Introduction

Mathematics revolves around conceptual dissemination and skills attainment and enhancement. This enables learners to acquire new mathematical skills and knowledge or concept to consolidate the old ones (Luneta, 2008). Teachers in schools use different teaching methods such as group work, exposition and explanatory, problem solving, practical work, direct instruction. Learners are evaluated on their understanding of the concepts in mathematics and this helps to reveal learners' errors (Borasi, 1994; Riccomini, 2005). Mathematics teachers in some schools like the ones participated in this study did not treat learners errors made in solving mathematical problems seriously as they confirmed in the casual discussion.

Arising from this casual discussion with the teachers who participated in this study about the learners' performance in mathematics, it was concluded that there was a need to analyse concepts that seemed to pose challenges. I started by looking at simple concept like grade 11 quadratic equations and teachers identified to me that learners had a serious problems in solving quadratic equations by completing a square. The poor performance of learners motivated me to think more about the types of errors learners displayed in solving quadratic equations by completing a square. In subsequent discussion with the teachers, we decided to give learners a diagnostic test to understand those errors in solving quadratic equations by completing a square. This paper describes the types of errors the grade 11 mathematics learners grappled with and why those errors are displayed in solving quadratic equations by completing a square.

2. Conceptual Framework

2.2.1 Conceptual and procedural knowledge

Conceptual knowledge is knowledge about the facts, concepts or principles upon which something is based (Microsoft Office Dictionary, 2001). Herbert and Lefevre (1986: 3-4) define conceptual knowledge as 'knowledge that is rich in relationship, that can be thought of as a connected web of knowledge, a network in which linking relationships are as prominent as the discrete pieces of information'. Such knowledge is described as that which is interconnected through relationships at various levels of abstraction (Confrey, 1990). It plays a more important role in the learning of mathematics than procedural one. It is essential for learners to have conceptual understanding, as in the absence of which they will ineffectively indulge in problem solving and follow wrong procedures to solve them (Centre for Develop Enterprise, 2007). Zemelman, Daniels, & Hyde (1998: 89-90), state that 'without true understanding of the underlying concepts, serious problem [guaranteed] with learning other concepts'. Focusing on understanding mathematical ideas makes students far more likely to study mathematics voluntarily and acquire further skills as they are needed (Zemelman et al., 1998)

Teachers should know their learners' mathematical thinking to be able to structure their teaching of new ideas to work with or correct those ways of thinking, thereby preventing learners from making errors (Sorensen, 2003). The way learners think about a concept depends on the cognitive structures learners have developed previously (Battista, 2001). Battista (ibid) also indicates that if learners cannot develop concepts by themselves, they will have a narrow understanding of those specific concepts, and will not be able to engage themselves in problem solving. Learners who do not have background knowledge in mathematics usually display numerous errors in solving mathematical problems, and this therefore results in most of learners grappling with quadratic equations by completing a square. Conceptual knowledge works hand in hand with procedural knowledge, the two complement each other.

Procedural knowledge should be taught in mathematics to reinforce understanding of mathematical concepts. Procedural knowledge is the ability of learners to use the relevant procedures in solving mathematical problems by following the rules, methods and procedures in different representations (Kanyaliglue, Ipek, & Isik, 2003; Hiebert & Lefevre, 1986). Procedural knowledge is a particular type of knowledge that learners display in solving problems and also in adhering to certain instructions when completing different tasks (Hiebert & Lefevre, 1986). Luneta (2008) asserts that procedural knowledge is specific to a particular task, and this implies that some procedures are not appropriate to solve certain mathematical problems. Learners may grasp relevant procedures but fail to use them correctly in solving mathematical problem (Siegler, 2003). Learners who lack this understanding are frequently using wrong procedures and this generates systematic patterns of errors in solving problems. Accordingly, teachers should not focus only on fact errors, but also on sources errors, especially when learners are making the same procedural errors (Gernett, 1992; National Research Council, 2002; Stein et al., 1997).

Riccomini (2005) explains instructional focus on facts as being problematic to teachers teaching parts of concepts or parts of procedural steps because teachers are trained to teach mathematics in terms of general concepts. This approach could help in addressing the problem of learners solving quadratic equations by completing a square. Riccomini also states that teachers' ability to recognise error patterns can be improved and that it might be possible to plan instruction that can help to alleviate problematic patterns in this concept. Measures to this effect might be effected through pre-service programmes, professional development opportunities in mathematics and refining curriculum material.

There is a relationship between procedural knowledge and conceptual knowledge. The correlation is shown when a learner is able to execute the procedures correctly and this displays a good grasp of conceptual knowledge (Siegler, 2003). If a learner has both procedural and conceptual knowledge, s/he can solve more complex problems of the same concept and provide justification of his/her solution to any given problem to solve. Learners with conceptual understanding only produce substantial gain in in both kinds of knowledge, but those with procedural understanding produce substantial gain in procedural knowledge but less in conceptual knowledge, which will ultimately impede the learners' growth in mathematics. Nesher (1986) supports the view that if a learner can only be shown procedures of solving a particular problem without understanding the concept, it is very unlikely that such a learner would be in a position to solve more complex problems independently. If problems are difficult to solve, then learned procedures may not help and will need a learner to have conceptual knowledge to solve them Nesher (1986). Conceptual knowledge also provides and constraints those procedures to be followed in solving mathematical problems (Confrey, 1990).

2.2 Issues on errors

Hansen et al. (2006: 15-16) define errors and misconceptions 'errors are mistakes learners make when solving problems that may be caused by carelessness, misinterpretation of symbols or text; lack relevant experience or knowledge related to that mathematical topic, learning objective, concept; lack of awareness or inability to check the answer given; or the results of misconceptions. Misconception is the misapplication of a rule, an over- or under-generalisation, or alternative conception of the situation'. The issue of misconceptions emerged here as errors are caused by misconceptions, but this study does not focus on misconceptions. Error discourse began as long time ago as the 17th century for instance, the work of Bacon in 1620 and also Pierce in 1887 cited in Luneta 2008. Learners make errors in some situations without realising them and if those errors are not recognised or dealt with, those errors can lead to more errors (Pickthorne, 1983). Pickthorne (1983: 1285) also asserts that too often teachers appear not fully realise the extent or the nature of their learners' confusions. Those teachers are unable to discover the reasons why those errors are made by the learners.

When we diagnose mathematical difficulties, we determine the areas of weaknesses a learner has, we study the specific errors the learner is frequently making, and we attempt to explain why those errors are being made (Troutsman & Alberto, 1982). Many teachers identify learners' errors but rarely diagnose them (Luneta, 2008). When analysing and diagnosing errors leaners display, we identify the root cause of those errors and how best they can be corrected in order to benefit both learners and teachers. Learners will not commit same errors and teachers will not teach same concept in the same manner. Teachers' mistreatment of errors and misconceptions has exacerbated their effects and has retarded learning (Pickthorne, 1983).

2.3 Methods of completing a square

To use methods of solving quadratic equations by completing a square, according to Laridon et al. (2010), learners should ensure that the coefficient of x^2 is 1 and if it greater or less than 1, they should divide by that coefficient, $ax^2 + bx + c = 0$. Learners should divide by a throughout before they could find the additive inverse of c both sides to have $x^2 + \frac{b}{a} = -\frac{c}{a}$. Learners can then add half the coefficient of x both sides before the equation could be factorised, $x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$. The equation can then be factorised on the left hand side and be simplified on the right hand side (Pretorias et al., 2006). The factors on the left-hand side are $(x + \frac{b}{2a})(x + \frac{b}{2a})$ and the right-hand side would be $\frac{-4ac + b^2}{4a^2}$.

2.4 Research objectives and questions

The study sought to explore the errors learners displayed in solving quadratic equations by completing a square. It also intended to respond to the following objectives:

- To identify and diagnose the types of errors learners displayed in solving quadratic equations by completing a square.
- To establish learners' perceptions about the sources of those errors.

The research questions follow the research objectives of this study:

- What types of errors learners’ possess in solving quadratic equations by completing a square and why?
- What are learners’ perceptions about the sources of those errors?

3. Research design

The study used a diagnostic test design followed by focus group interviews to understand the reasons behind the errors learners had in solving quadratic equation by completing a square. It was conducted under qualitative research paradigm in which the information collected was analysed through description and not statistically (Rule & John, 2011). The study was conducted in (N = 5) secondary schools of same circuit which comprises of 11 secondary schools. The total population of all the learners in the five schools is one-hundred and ninety four learners (N = 194). Learners in the three schools are taught by three male teachers and the other two schools learners are taught by female teachers. The teaching experience of all the teachers range between 5 – 10 years and their ages range 25 – 37. The focus of the study was on diagnosing errors learners had in solving grade 11 quadratic equations by completing a square. Learners’ scripts (N = 65) were sampled from (N = 5) schools, in which scripts were randomly sampled from each school (N = 15) and (N = 10) learners participated in the focus group interviews which gave a total of 50 learners to explore the sources of the errors and misconceptions. Each group of learners were represented by (N = 5) female learners and (N = 5) male learners to observe gender equality. I also wanted to find out the root causes and reason why those errors occurred in solving those equations by completing a square.

3.1 Instrument

The instruments used in this study were the diagnostic test and semi-structured focus-groups to collect data pertinent to this study. The diagnostic test comprised of eight equations with the total mark of 40 and each equation was allocated 5 marks each. The test assisted in identifying and diagnosing the types of errors learners had in the five schools in solving quadratic equations by completing a square (Salvia & Ysseldykes, 2004). The focus-groups helped me in understanding the reasons why leaners displayed those errors in solving quadratic equations by completing a square. The collected data through diagnostic test and focus groups attempted to respond to the following objectives and the research questions of the study.

4. Findings and discussions

The findings and discussions are divided into two components in which the first one dwells on the types of errors identified in solving quadratic equations by completing a square. The second component dwells on the learners' perspectives on the sources of errors they displayed.

Component 1: Types of errors identified

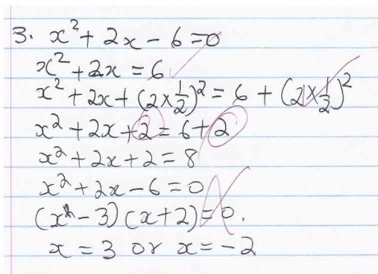
The first component of the analysis of learners' scripts in solving quadratic equations by completing a square was to identify the errors learners made. The collected data was analysed by identifying the common errors learners displayed in solving equations by completing a square. Those common errors were characterised by conceptual errors and procedural errors. The table below characterises tasks, steps, errors learners made in solving some of quadratic equations by completing a square given to learners in a test.

Table 1: Error identification

Task	Steps	Error
$x^2 - 2x - 1 = 0$	- Finding additive inverse both sides - Adding half the square of the coefficient of x - Multiplying terms in the inner brackets - Adding like terms on the right-hand side - Factorising the left-hand side - Removing squares on the left-hand side - Determining the value of x	- Most of the learners failed to find the additive inverse of -1. - Added $(2 \times \frac{1}{2})^2$ on the left-hand side only. - Multiplied 2^2 before multiplying by $\frac{1}{2}$ and got the answer 2. - Factorised without notifying - Wrong x values
$2x^2 - 2x - 9 = 0$	- Divide by 2 throughout. - Additive inverse.	- Failed to divide by 2 throughout to get

	- Add half the square of the coefficient of x (the other remaining four steps indicated above)	$x^2 + x - \frac{9}{2} = 0$. - Unable to find the additive inverse of $-\frac{9}{2}$ both sides to get $x^2 + x = \frac{9}{2}$. - $x^2 + x + (1 \times \frac{1}{2})^2 = -\frac{9}{2}$ - $x^2 + x + 1 = -\frac{9}{2}$ - $(x + 1)(x + 1) = -\frac{9}{2}$
$x^2 - 4x = 0$	- Adding half the square of the coefficient of x - Solving terms in brackets - Factorising the left-hand side	- Added half the coefficient of *x* on the left-hand side only and square the coefficient of *x* before multiplying by $\frac{1}{2}$ and got the answer as 8. - (x - 4) (x + 2) and the values of x are 4 or -2.
$x^2 + 2x = 6$	- Adding half the square of the coefficient of x - Solving terms in brackets	- Also squared the coefficient of *x* before multiplying by $\frac{1}{2}$ and got 2 as the answer. - Most of them got factors as $(x + 1)(x + 2)$ and the value of was found as -1 or -2.

Most of the common errors found were dividing by the coefficient of x^2 if the equation was greater than 1 or less than zero (Laridon et al., 2010). Moreover, some of the common errors were the ones of simplifying $\left(2 \times \frac{1}{2}\right)^2$ from the equation, $x^2 - 2x - 1 = 0$. Most of them got the answer as 2 and pursued them by the focus groups to understand why they solved the problem in that way. Other learners did not find the additive inverse of a constant -1 before completing a square, which was also wrong for them to solve the equation in that fashion. More errors were found when some of them were failed to factorise the equation after completing a square which revealed that learners lacked knowledge of factorisation. Zemelman et al. (1998) asserts that learners with a narrow understanding of a concept lead her/him not to understand other related concepts. The sample below represent the one of the scripts sampled which displays common errors learners made during the diagnostic test.

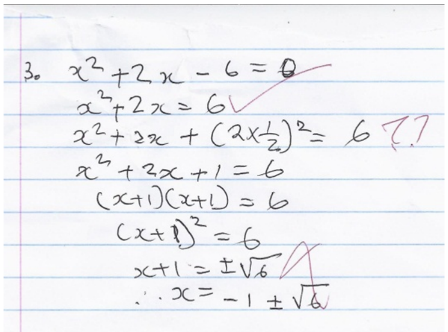


Learner's sample 1

Description of the sample 1

The sample was taken from one of the scripts I have sampled to identify and diagnose errors learners displayed in solving quadratic equations by completing a square. This equation was sampled from the scripts that had displayed similar errors in the quadratic equations given to learners to solve. The learner has successfully added half the coefficient of *x* both sides, but failed to simplify the terms $(2 \times ½)^2$in the brackets in which the learner squared 2 first and found the answer as 4 and multiplied the answer by ½. The learner thought that for any expression with a certain power should be simplified by starting with term outside the bracket. This was the other error learners had displayed in solving quadratic equations by completing a square. The learner then wrote the equation in standard form $x^2 + 2x - 6 = 0$, in which a wrong procedure was followed instead of factorising the left-hand side and introducing the square root both sides. The wrong factors were given and the learner confirmed that from these factors (x - 3) (x + 2) when you multiply *x* by *x* the answer is x^2 and when multiplying -3 and +2 the answer is -6. She did not consider the middle term when multiplying terms in brackets, but only considered the first term and the last term which led her to the wrong answer. She should have multiplied 2 by ½ and got the answer as 1 before squaring her answers in the brackets both sides. The new equation would have been $x^2 + 2x + 1 = 7$, and the equation was not supposed to be written in standard form according to

Laridon et al. (2011). The factors should have been $(x + 1)^2 = 7$ and introduce the square root both sides to determine the value of x as $x = -1 \pm \sqrt{7}$.



Learner's sample 2

Description of sample 2

This was sampled from one the learners' scripts which displayed similar errors in solving the given quadratic equation by completing a square. He (boy) has successfully found the additive inverse of -6 and +6 both sides before completing a square. The square was only completed one side not both sides of the equation. The terms in brackets were correctly simplified in which 2 was multiplied by ½ and the got the answer as 1. The new equation was $x^2 + 2x + 1 = 6$ and found factors as $(x + 1)(x + 1)$ which were correct, the only problem was adding half the coefficient of *x* on the right hand side to get the answer as 7 (which is 6 + 1). This learner has the knowledge of solving quadratic equations by completing as evidenced in the sample. Siegler (2003) asserts that learners who grasp procedures normally fail to follow them in solving mathematical problem. But most of the learners' scripts showed that learners did not have knowledge of solving quadratic equations by completing a square as they failed to follow the correct steps after adding half the coefficient of *x* on the left hand side only. These learners display a lack of knowledge, the lack of awareness (Hansen et al., 2006), of solving quadratic equations by completing a square. They had incorrectly simplified terms in the bracket, the coefficient of *x* was first squared and the answer 4 that was found multiplied by ½ which has given the answer as 2. Most of those learners grouped the terms and the equation was written in standard form, $x^2 + 2x - 4 = 0$. Some of the learners found their factors as $(x - 2)(x - 2)$ *and* $(x + 4)(x - 1)$. Some of those learners (N = 10) who gave these factors were interviewed and said that when they multiply the first terms they get the answer as x^2 and when they multiply the last term they get the answer as -4. All the learners did not consider the fact that when they remove the brackets, all the terms are supposed to be taken into consideration. Battista (2001) concurs that the way learners think depends on what they have learnt previously. These learners showed that they lacked knowledge of simplifying terms in brackets and factoring.

Others rewritten their equations in the form of $ax^2 + bx + c = 0$ as shown in sample 1 step 3 from the bottom, which was incorrect and then factorised instead of factorising the equation on the left-hand side without writing it in standard form. Some of them found the additive inverse of the equation but only completed a square on the left hand side and failed to do it on the right hand side which had led learners to the wrong answer. Another common error was that some of the learners had failed completely to attempt to complete a square in solving quadratic equations, such as dividing the coefficient of x^2 if was greater or less than 1. Laridon et al., (2011) advise that when adding half the coefficient of x, learners should ensure that the coefficient of x^2 is 1. They further stated half the square of the coefficient of x should be added both on the left hand side and the right hand side. This view was supported by Zemelman et al., (1998) that learners without true understanding of the underlying concepts guarantee serious problems in learning other concepts. This is what happened to these learners as revealed that most of them were unable to solve quadratic equations by finding factors and this had continued on to this one of completing a square. Siegler (2003) also asserts that a learner without a good understanding of a concept result in using procedures of solving problems inappropriately. Most of these learners did not know how to solve the equations and showed that specific knowledge of procedures was not acquired by those learners. Luneta (2008) supports that procedural knowledge is specific to a particular task and if that knowledge is not learnt then it is difficult for learners follow that procedure. If a learner does not understand the concept, it would be difficult for him/her to articulate the procedures to solve problems based on that particular concept (Battista, 2001).

Component 2: Error diagnosis

In diagnosing errors, I understood the reasons why the leaners had those errors in solving quadratic equations by completing a square. The reasons behind those errors were found by using focus group interviews in order to understand the sources of those errors.

Table 2: Sources and justifications of errors and misconceptions

Source	Justification
1. Lack of knowledge of additive	The knowledge of applying additive inverses

inverse	of terms was not displayed as most of the learners was supposed to determine the additive inverse of a constant. They misapplied the rule that the additive inverse of a term is takes the opposite sign of that term.
2. Knowledge of simplifying terms in brackets	Most of the learners failed to simplify terms in brackets, instead they did other way round. Lacked knowledge of dealing with the inner bracket terms before they could simplify the whole expression. They squared the coefficient of x before they could multiply it by half. This is the misapplication of a rule, completing half the coefficient of *x*.
3. Lack of understanding of making the coefficient of x^2 be 1	Learners intertwined solving quadratic equations by factoring and using quadratic formula with this method of completing a square where the coefficient of *x* should always be 1. This led learners not to follow the correct procedures in solving quadratic equations by using this method.
4. Factorising the terms in completing a square	Most of the learners did not know that the factors should always be the same in the two brackets and be multiplied to give a square bracket.
5. Knowledge of adding half the coefficient of x both sides	Knowledge of balancing equations was not displayed in most of the learners as they only added half the square of the coefficient of *x* on the left-hand side and had left the right-hand side.
6. Lack of knowledge of solving quadratic equations by completing a square in general	It was difficult for most of the learners to show an understanding of solving quadratic equations by using this method as some

	failed to start solving equations. The types of errors displayed by the learners showed that they did not understand how to solve quadratic equations by completing a square.

In focus groups, some indicated that this concept is challenging to them as they compared with factorisation and using quadratic formula. In the discussion, learners revealed that some of their teachers don't give them an opportunity to participate and only the fast learners are always given a platform. In some schools learners are unable to participate as their teachers praise those who give correct answers. Those learners were again unable to participate as they were afraid of being mocked at by their fellow learners. Learners also blamed themselves for not being committed to their school work due to their laziness, some mentioned that they had parental role to play at their homes. It was also revealed that the approaches teachers used to teach mathematical concepts were difficult to understand. They said some of the teachers were fast and don't give learners chance to engage with them. It was also indicated that other teachers were not committed to their work, like coming to class late and not controlling their work. From the focus group interviews, most of the learners were not given an opportunity to discuss the concepts and some indicated that they did not have enough time to practice their work.

5. Conclusion Remark

It is imperative for teachers to teach mathematics using learners' errors and misconceptions as this will guide them on what learners grapple with. They will be able to use multiple strategies or approaches to teach mathematical concepts to cater all the learners including the slow ones. In this study learners were unable to divide by the coefficient of x^2 if the equation was greater than 1 and less than zero, multiplying by half the coefficient of x and also additive inverses of any constant given. Some reasons which contributed to these difficulties were: learners' laziness, learners' lack of participation, teachers' teaching participation, teachers' commitments and conduct towards learners; and peers' behaviour in the classroom. Teachers are not supposed to identify learners' errors learners make in mathematics including quadratic equations by completing a square, but they should also diagnose them to understand learners' challenges.

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