

Effective Instructional Approaches Used by Teachers to Teach Grade 11 Quadratic Equations in a Context of South African Schools of Limpopo

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KEYWORDS Teachers' teaching instructions and methods, quadratic equations, factoring, completing the square and quadratic formula.

ABSTRACT This paper discusses the effectiveness of teachers' teaching instructional approaches used to teach Grade 11 quadratic equations. Four teachers and the researcher's colleague were observed teaching quadratic equations by factoring, completing the square and using quadratic formula as was the area of focus of the researcher. Questionnaires were used to interview these teachers and their five learners in each school. The purpose of observing those lessons was to identify the types of instructions they used to teach quadratic equations by using these three methods. The instructional approaches used by the teachers were similar to what the researcher used before the study. The intervention strategies were used by the researcher to alleviate the difficulties learners experienced in solving equations. The instructional approaches used by the researcher were deemed to be effective as most learners performed well after the interventions. The study is anchored within qualitative research invested in action research which has taken place in the classroom.

INTRODUCTION

The National Curriculum Statement (NCS, 2005) requires that teachers implement the curriculum effectively in the classroom. Teachers have to apply the rules and principles of

NCS appropriately so that learners do not experience problems in acquiring the new learning area and the outcomes of the curriculum. However, learners are experiencing difficulties in solving mathematical problems (Center for Education and Development Enterprise, CDE 2007). This was documented in the Trends in International Mathematics and Science Studies (TIMSS 2003) which revealed that most of learners at Grade 8 in South Africa are not performing well in Mathematics.

Researchers (Adler 2004; Luneta 2008; Setati 2005) had documented some reasons for errors that learners display and misconceptions they possess which contributed to their poor performance. These include:

- Teachers' instructional strategies or questioning styles.
- Students' various styles of learning.
- Different levels of cognition (the concept is much higher than the cognitive level of understanding of learners), and
- Learners lack of procedural or conceptual knowledge.

The purpose of this research was to study the teachers' teaching instructions used to teach Grade 11 quadratic equations by factoring, completing the square and using quadratic formula. Learners in the researchers' school performed poorly in grade 11 mathematics for four consecutive years, 2009 – 2012. The poor results in mathematics prompted the researchers' curiosity to investigate problems learners faced in solving mathematical problems. The problem was diagnosed through examination results in grade 11 mathematics, class tests, class activities and home activities in the researcher's school. In analysing the examination results, the researcher had identified that learners had performed poorly in Mathematics for four consecutive years. The analysis of results also showed that learners performed poorly in quadratic equations where the researcher had expected them to score high marks. This, therefore, dropped the average percentage of grade 11 mathematics results to unprecedented low level. The problem experienced had prompted the researcher to investigate the difficulties learners faced in solving quadratic equations and teachers' instructional approaches used in teaching this concept.

THEORETICAL FRAMEWORK

The researcher followed a constructivist learning theory (Piaget 1970; Skemp 1979) which asserts that concepts are not taken directly from experience. It considers learners' prior-knowledge in teaching and learning. This theory does not view learners as empty vessels

where teachers transmit knowledge into learners' minds, unlike behaviourist or connectionist learning theory in which learners learn what they are taught. Learners have been given an opportunity to interpret, organise and structure mathematical concepts during the process of teaching and learning of quadratic equation, called schemas.

This learning theory basically plays a fundamental role in researchers' and teachers' work, in observing learners' current schemas and new ones in quadratic equations. Therefore, this theory is in line with what the researchers wanted to engage themselves in this study. Observations are driven by the pre-existing knowledge. Learners' experience is taken into account in teaching and learning (Piaget 1970; Skemp 1979). Learners' current schemas and new ones are prerequisites in this theory and therefore guided the researcher in the teachers' instructional approaches used to teach quadratic equations. Teachers' and learners' engagement in learning quadratic equations concepts, i.e. factoring method, solving quadratic equations by using the formula and solving quadratic equations by completing a square were the area of focus. The two schemas (current and new one) interaction involved assimilation and accommodation. In constructivist theory, learners used existing knowledge to accommodate new ideas in solving quadratic equations.

Different teaching method in mathematics classroom

Teaching methods are particular techniques the teacher uses to help learners gain knowledge to achieve a particular learning outcome required by syllabus (Jacobs, Vakalisa, & Gawe 2004). Teachers need a well-planned procedure in using teaching methods to effectively guide and motivate learners in their learning. Teaching methods that are well-planned or effective enable learners to feel empowered that mathematics is personally useful and meaningful, and to feel confident that they can understand and apply mathematical concepts effectively in solving problems (Sorensen 2003). This is described as 'mathematical inspiration' developed by the teacher (Zemelman, Daniels, & Stein 2001), in which learners are developed with mathematical power. Teaching should be flexible and teachers are required to experiment with different teaching methods, such as discussion, questioning methods, expository and telling, and problem-solving methods. Using different methods in teaching mathematics in the classroom enhances effective teaching of mathematics.

The discussion method is one of the methods used and useful for learners as it is considered as one important prerequisite for effective teaching (Askew, Brown, Rhodes, Johnson, &

William 1997). Discussion is mostly involving cooperative learning, teacher-led discussion and whole class discussion. Battista (2001) asserts that learners' group discussion should be used with care, maybe assigning scribes, group leaders or maybe using jig-saw method (Siegler 2003). This type of discussions are characterised by dialogue in which the teacher and learners discuss by listening to each other (Hansen, Drews, Dudgeon, Lawton, & Surtees 2006), in finding solutions to quadratic equations. Class discussion takes place after group discussion and involves the teacher and learners. It is effective to provide closure to mathematical problems (Grouws & Cebulla 2000). Reynolds and Muijs (1999) asserts that some research shows that class discussion and teacher-led discussion are the most effective mathematical instructional methods in teaching basic skills. These methods cited above were most important in addressing the challenges learners faced in solving quadratic equations by using three methods, factoring, solving by using quadratic formula and completing the square.

Teachers and learners also interact through questioning and telling methods. Telling method is the one that involves the teacher as the main imparter of information while learners are passively listening (Pimm & Johnston-Wilder 2005). It is practiced when giving the instruction and explaining, and also focuses on what has to be done or still has to be done during teaching and learning. Pimm and Johnston-Wilder outline two forms of telling methods, as those of exposition and explanation. Explanation takes place from what learners currently know to a point beyond this where the teacher would like them to expand their knowledge. And, exposition is when the teacher talks directly to learners from their own mathematical thinking. Exposition required the teacher to use with care as too much exposition might cause some learners lose concentration (cf. Pimm & Johnston-Wilder 2005).

Questioning is another technique used to interact with learners in mathematics classroom to enhance teaching and learning of quadratic equations. Hansen et al. (2006) describe the learning of mathematics as taking place within a social situation of talks, compromising of discussion, prompts and answers in which the teacher and learners are as much a part of what mathematical activity is as what is in the textbook, on the worksheets, or the chalkboard. Questioning techniques is another method used by teachers asking learners questions to develop understanding of what they know about the concept not with the intent of getting the correct answer, but to enhance learning (Hansen et al. 2006; Cruikshank, Jenkins, & Metcalf 2009). It should encourage reasoning and allow learners to draw from their prior knowledge

rather than accepting 'yes or no' answer (Cooper 2010). It is dangerous for mathematics teachers to focus on the responses that learners thought were acceptable, avoiding suggested alternatives and hidden queries or areas of confusion in teaching quadratic equations. Teachers relying on the correct answers are likely not to detect underlying misconceptions. Effective questioning should increase class participation and it enhances learners' learning experience (Critelli, Tritapoe, & Odhiambo 2010). These methods were carefully used to assist in teaching and learning of quadratic equations by using the three methods.

Learners should also be taught on how to solve mathematical problems. Problem solving is the interaction of cognitive process and mental representation in diverse ways (Green & Gilhoolly 2005; Halpern 1997) and it is a complex activity. In problem solving, a learner has to describe a problem in her own way to show understanding of mathematical concepts, which was what the researchers done guided by this literature to teach quadratic equations. Luneta (2006) indicates that a problem solver should understand the problem in order for him/her to analyse it, breaks the problem into smaller chunks, execute the problem and reflect on the action taken. Sternberg and Benzeev (2001: 145), state two types of problems, namely, well-structures and ill-structured problems: well-structured problems are 'those that have at least one specified path to solution, whereas ill-structured problems lack such specificity'. Steiner (2006) used complex problems instead of ill-structured ones, which have more elements and require greater interaction among those elements and more sophisticated methods than simple problems. Learners should be taught how to be engaged in problem solving as long as there are no personal or external factors that hinder the learning process (Green & Gilhoolly, 2005; Halpern 1997).

METHODOLOGY

The study was qualitative research under action research which investigated problems pertinent to the researcher's work and learners' work in his school. According to Zuba-Skerrit (1996), action research is any research that is systematic enquiry either by teachers, principals, school councillors, or any other school stake holders in teaching and learning environment to gather information about: ways in which their particular school operates, how they teach, and how well their learners learn.

A precondition for undertaking action research in the area of focus was for the researcher to organise and support the learners in order to produce good results. This had also helped to

alleviate the errors and misconceptions learners experienced in solving quadratic equations. In alleviating those errors and misconceptions, the researcher had employed the intervention strategies adopted from the literature review. The intervention strategies had been used for three consecutive years, 2008, 2009 and 2010.

The researcher had given learners a pre-test to select the sample for his research. The school had 84 Grade 11 learners of which all of them wrote the test. Learners who performed lowly were the ones who were purposefully sampled to participate in the study. The researcher did a self evaluation by teaching learners quadratic equations by using the three methods. One had used question and answer method, exposition, explanation, discussion method, whole class and teacher led discussion. The following concepts were taught during the study, finding common factors and linear equations before quadratic equations could be introduced. These methods were used in 2009 and 2010 to gauge their effectiveness in teaching and learning.

Data Collection

Data were collected from four schools in the Limpopo province of South Africa. It was made up of observations of teachers as they taught quadratic equations. The researcher had visited each school three times for lesson observations. The study purposefully sampled five teachers that agreed to take part. The five teachers with their five learners per school were interviewed using a developed semi structured schedule. Learners' scripts on quadratic equations were reviewed for errors they displayed and the misconceptions they possessed. Teacher were pseudonym named as teacher A, B, C, D, and E according to schools also named as 1, 2, 3, 4, and 5 respectively.

School	Observations date	Name of teacher	Gender	Area	Area of focus
1	18/20/22-08-08	A	Male	Rural	Quadratic Equations
2	18/20/22-08	B	Male	Rural	QE
3	26/27-08 & 02/-09	C	Female	Urban	QE
4	14/18/27-08	D	Male	Rural	QE
5	12/13/20-08	E	Male	Urban	QE

The purpose of lesson observations was to learn more about the instructional approaches used by the five teachers when teaching quadratic equations and benchmarked it against the predetermined instructional approaches used by the researcher to teach quadratic equations. The researcher used approaches that are recommended as effective instructional approaches from the literature (Johnson & Christensen 2004).

Data analysis

Data in this research were analysed by using descriptive narrative (Johnson and Christensen 2004). This was to render a description of teachers' teaching approaches and learners' work based on class tests, pre-test and post-test. The intervention strategies used by the researcher adopted from the literature were compared and contrasted with the methods used by the five teachers. Data analysis is the breaking down of raw data into more understandable information (McMillan & Schumacher 1997). Analysis of the lessons was done by using Indicator Evaluation Forms for Classroom Instructions adopted from Luneta (2011). Teachers' teaching methods were referred to the question and answer method, problem solving, exposition, explanation and discussion method (group discussion, teacher-led discussion and whole class discussion). Teachers' teaching approaches and teaching methods were analysed singly as follows.

Question and answer method

This teaching method was previously not used to develop and enhance learning before the researcher adopted the informed one from the literature review. The researcher before the intervention strategies had asked learners questions to get correct answers as was what other teachers had done in using this method and this is contrary to Critelli et al. (2010) that it should encourage learner participation. One had changed to choose learners randomly to identify the errors learners had in the topic concerned. The researcher wrote the equation on the chalkboard, $x^2 - 3x - 4 = 0$ and asked learners to give the type of the equation. He chose one of the learners who raised his hand. This type of questioning method had disengaged other learners, especially the slow ones, in teaching of quadratic equations and other mathematical concepts. The teachers who participated in the study had used the uninformed method of questioning learners to get correct answers instead of developing and enhancing learning in quadratic equations (Hansen et al. 2006; Cooper 2010; Cruickshank 2009). This had impeded learning in quadratic equations as teachers did not identify the underlying errors

and misconceptions learners possessed. These teachers had worked with learners who raised their hands, mostly the high achievers. Questions like what are the factors of the equation were asked by all the teachers and answered by learners who raised their hands. This had contributed to errors learners had made in solving quadratic equations and hence other Mathematical concepts. The questioning technique adopted from the literature had assisted the researcher in identifying and addressing the errors and misconceptions learners had in solving quadratic equations.

- Telling method

The telling method involves the teacher as the main imparter of information while learners are passively listening to their teacher (Pimm & Johnston-Wilder 2005). Previously the method was not used with care and contributed to learners errors made in solving quadratic equations and other Mathematical concepts. This method was used by the researcher and the five teachers who participated in the study in teaching quadratic equations. The researcher had refrained from using this method after he adopted the informed one from the literature which made him to apply it in teaching and learning of quadratic equations and other Mathematical concepts. Teacher A, B and C had used telling method but were much better as compared to D and E. Teacher D and E had done most of the talking and interacted with learners for a short time.

Pimm and Johnston-Wilder (2005) had outlined two forms of a telling method as exposition and explanation. Exposition is when the teacher talks directly to learners and learners are passively listening to what their teacher is delivering to them. Explanation is when the teacher takes learners from their current knowledge position to the new one. The researcher had used exposition in conjunction with explanation to reinforce understanding in teaching and learning of quadratic equations. The researcher had previously started his lesson by saying “class today I want us to treat quadratic equations by factorization and defined the equation $ax^2 + bx + c = 0$ ”. He then showed learners how to find factors of quadratic equations by factoring.

The observed teachers had used the same method of explanation to reinforce understanding in teaching and learning of quadratic equations except Teacher D and E. The two teachers had mostly used telling method which showed learners how to factorise quadratic equations, how to complete the square and using quadratic formula which had made some learners to

lose concentration during teaching and learning. This study had brought some changes as the researcher had applied the informed approach to address the difficulties learners experienced in solving quadratic equations and other Mathematical concepts. Telling method is effective when teachers use it with care to avoid learners to lose focus during teaching and learning. It has to be used when learners are passive during the lesson.

Problem solving

The researchers had engaged his learners in problem solving to solve quadratic equations by factoring, completing the square and using quadratic formula. Learners were given problems to solve in groups as to develop and enhance learning of quadratic equations and other mathematical problems. Problems such as $x^2 - 3x - 4 = 0$ were given to learners to solve in groups and some of the learners grappled with them and the researcher helped them finding the factors. Problem solving is effective when it is used in learners' group discussion as creative thinking, critical thinking and argumentation are encouraged. Teacher A, B and C had used this teaching method but was not so effective in solving quadratic equations by completing the square even after the researcher had used intervention strategies. They also gave their learners problems to solve in groups except Teacher D and E. Mathematics teachers need to be encouraged to use problem solving in their teaching and learning to enhance and develop classroom learning.

DISCUSSION METHOD

The discussion method is the interaction either between the teacher and learner or learners amongst themselves (Sorensen 2003). Discussion method in teaching and learning of quadratic equations had involved learners' group discussion, teacher-led discussion and whole class discussion. Previously the researcher had used group discussions that were unmonitored which disengaged the low achievers as high achievers were the ones who participated mostly in their groups. The researcher arranged his learners without giving them tasks to do in their groups (Battista 2001). This had an impact on learners' performance in quadratic equations and hence other Mathematical concepts. The only way to avoid the formation of entrenched misconceptions is through discussions and interactions (Hansen et al. 2006). Teacher-led discussion was useful for learners' learning and was considered as important prerequisite for effective teaching (Askew, Brown, Rhodes, & William 1997). The researcher had used this type of discussion before he could use group discussion as it gave learners directive on what had to be done in solving quadratic equations.

In learners' group discussions, one had used two types of group discussions, home and family group discussions which is called the jig-saw method (Siegler 2003). In home group discussions learners were labelled as number 1, 2, 3, *et cetera* and each same labelled learners form family group and solved the problem of their label such as family group 1 solved problem 1. Learners were given problems such as $x^2 + 4x = -2$ to discuss in family group and then home group.

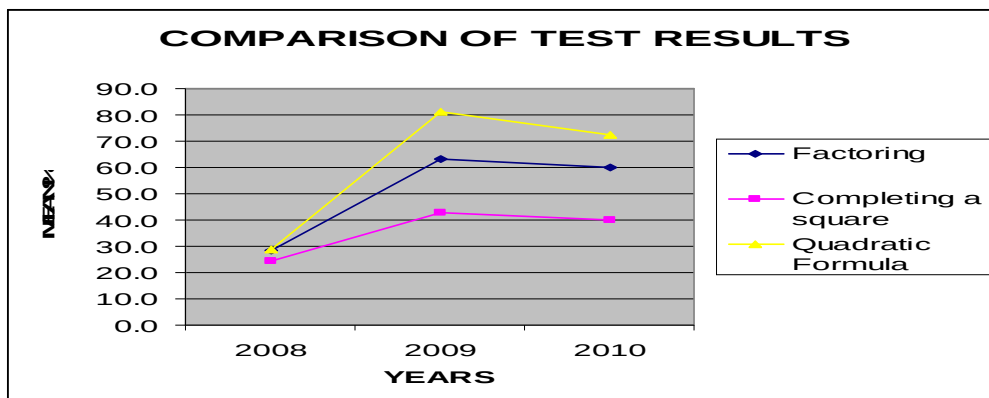
This type of group discussion was different from what the observed teachers had used in teaching and learning quadratic equations except Teacher C who applied the same method of discussion. Teacher A and B had used unmonitored group discussions which had disadvantaged some learners especially the slow ones as they could not participate in the discussions. This was what the researcher had done before the study. He conducted his lessons in this manner but realised after reading the literature that it was ineffective. He developed the intervention strategies and "better" ways of teaching quadratic equations from the literature review. The teachers group discussions were not well organised as learners were mostly not assigned tasks such as scribes, reporters, leaders or captains to perform in their groups (Battista 2001; Sorensen 2003). Teacher D and E did not use any group discussion whatsoever.

The learners' group discussions mostly lead the teacher and the learners to whole class discussion. Whole class discussion was especially appropriate towards the end of the lesson as it provided the researcher and Teacher A, B and C to the closure or conclusion of the concept (Grouws & Cebulla 2000). In this type of discussion, learners and the researcher had reached common agreement about how to find factors of equations by using the first term and the last term with appropriate signs in between terms. They also concluded that in solving quadratic equations by completing the square they added half the square of the coefficient of x and lastly in using quadratic formula the value of a , b and c should correctly be identified with their signs per se and substituted in the formula.

The types of errors displayed and misconceptions possessed by learners in solving quadratic equations by using the formula were addressed by the researcher who used intervention strategies adopted from the literature. The researcher had emphasised that learners understand how to write the equation in standard form $2x^2 + 8x = -3$, to be $2x^2 + 8x + 3 = 0$. One gave learners more equations that were not written in standard form for them to solve so that they

could be exposed to them. Learners had turned to be able to use the correct formula and written the equations in standard form ($ax^2 + bx + c = 0$). The value of a , b and c were correctly identified and substituted in quadratic formula. The interventions had played a proactive role in alleviating the difficulties learners had had in solving quadratic equations by using the formula. The mean percentage of solving quadratic equations by using quadratic formula after intervention strategies was much higher as compared to factoring and completing the square.

The researcher had used the graph to compare the results of pre-test with the short tests given to learners according to the stated sub-topics of quadratic equations. The pre-test results were also analysed separately according to the sub-topics as in the short tests. The graph that follows illustrates the comparison of tests results written in 2008, 2009 and 2010 to check the validity of the intervention strategies the researcher had used in teaching quadratic equations. The graph compares the test results for the year 2008, 2009 and 2010 based on factoring, completing the square and using quadratic formula.



The graph above had shown that the intervention strategies used by the researcher had played a fundamental role in teaching quadratic equations except in completing the square which had not made much difference in 2009 and 2010. There was a satisfying shift in learners' percentages in solving quadratic equations by factoring and using quadratic formula. In factoring, most learners were able to find factors of quadratic equations, allocated appropriate signs in between terms in brackets and had given the correct roots or solutions. Learners had also understood how to write the equation in standard form ($ax^2 + bx + c = 0$) as previously others did not write them in standard form especially when they applied quadratic formula.

Learners were given this problem, $x^2 = 12 - 9x$ and most of them identified the value of a , b and c without writing it in standard form, $x^2 + 9x - 12 = 0$ before they could solve it. Since the intervention strategies were applied by the researcher, they ensured that they have written the equations in standard form before they could identify the value of a , b and c . They had correctly used the formula, substituted the value of a , b and c in the formula and had successfully solved the equation to get the solution. Most of the learners had still experienced difficulties in solving quadratic equations by completing the square and this, therefore, had informed the researcher that the intervention strategies were deemed ineffective in teaching this concept. Learners were still unable to add half the square of the coefficient of x both sides, on the left hand side and right hand side of the equation. The errors they displayed and misconceptions they possessed in the pre-test were analysed singly, factoring, completing the square and quadratic formula under document analysis in the dissertation as this paper focused only in effective instructional approaches to teach quadratic equations.

CONCLUSIVE STATEMENT

Teachers' teaching instructions that were not informed by the literature used to teach quadratic equations by factoring, completing the square and using quadratic formula had contributed to learners' errors and misconceptions. These types of instructions were deemed to be ineffective in teaching quadratic equations by using the three methods and other mathematical concepts. The intervention strategies such as questioning, problem solving, exposition and explanation, and discussion methods that had been employed by the researcher had played a fundamental role in alleviating the difficulties learners had experienced in solving quadratic equations. They had previously not used them with care which resulted in contributing to learners' errors and misconceptions in solving quadratic equations and other concepts. These informed approaches had been used in 2008 during remediation, 2009 and 2010 were effective in teaching quadratic equations and other mathematical concepts. But it was ineffective in solving quadratic equations by completing the square as the learners had still experienced problems in solving these types of equations. The intervention strategies for teaching quadratic equations by completing the square still need to be investigated to alleviate learners' difficulties in this concept. It will be imperative for teachers to use these intervention strategies to teach mathematical concepts in their schools to enhance and develop teaching and learning. Research still has to be conducted in other methods of solving quadratic equations such as solving quadratic equations by squaring

both sides, solving equations with fractions, solving equations by substitutions and simultaneous equations.

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