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Strands of Mathematical Proficiences in Grade 12 Learners' Responses to Final Examinations
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KEYWORDS adaptive reasoning, conceptual understanding, mathematical proficiency, strategic competence

ABSTRACT This article presents the results of an analysis of learner's scripts that identified learner's strands of mathematical proficiencies displayed by grade 12 learners in response to final mathematics examination questions in the Gauteng Province in South Africa. The study focused on four topics namely, sequences and series, differential calculus, analytical geometry and trigonometry. The lens of strands of mathematical proficiency was used to unpack and classify learner's responses which were either procedural or conceptual knowledge. The analysis which was deductive came up with three categories of strands of learners' mathematical proficiencies, and these were, not proficient, moderately proficient and proficient in examination questions that demanded procedural and conceptual knowledge. The findings were that it was most likely that learners left high school without being proficient on these topics which incidentally formed the bulk of the prerequisite mathematics course in engineering and other natural science programmes at university.

1. INTRODUCTION

Grade 12 Mathematics achievement in the Republic of South Africa has been a concern irrespective of recent educational reforms (DoE, 2002). Low achievement in mathematics, physical science and accounting has been noted during the release of Matric results by the Department of Basic Education (DBE) in recent years(DBE, 2013; Mji and Makgato, 2006; Chen, 2014). This paper seeks to identify grade 12 learners'strands of mathematical proficiencies displayed in response to examination questions.

In this study the notion of mathematical proficiency was used as a lens to view learners' strands that grade 12 learners performed in their response to examination questions (Kilpatrick, Swafford and Findell, 2001). The 2003 TIMSS study showed that South Africa was the lowest in Mathematics achievement amongst the participating countries, (Leung, 2014;2005). A study by Taylor and Vinjevold (1999) that conducted observations in classrooms identified challenges in mathematics education in South Africa such as; domination of teacher centeredness, lack of teaching and learning structures that promoted higher order reasoning, prevalent of misuse of everyday examples by teachers in their teaching and low teachers' morale and self-esteem. However such challenges might influence the strands of mathematical proficiency that learner's exhibit which is the focus of this study. Kilpatrick et al. (2001) conceptualized procedural fluency and conceptual understanding as forms of mathematical knowledge whilst strategic competence and adaptive reasoning were seen asmathematical skills.In the context of this study these strands were in various ways identified in learners' responses to the final examination questions that were under consideration.

1.1 STATEMENT OF PROBLEM

According to Chisholm (1999) final examination's focus is on the following; test the knowledge of the syllabus, are used as a filter for university admission, assess learner's achievement of learning outcomes and also hold departmental officials and educators accountable for teaching and learning. However Sieborger and Macintosh (1998) aver that it is impossible to structure final examinations such that it is totally valid or reliable due to political issues developed by parents, employers, educational authorities and tertiary institutions. My reasoning on such high-stakes final examinations illuminates questions towards the strands of mathematical proficiency that learners exhibit in their responses such as; are the strands of mathematical proficiency that learners display in their responses to final examinations the determinant of students' success at the university level? How can the strands of mathematical proficiency be used as pointers to low mathematics achievement at high school?

A study by Stein, Grover and Henningsen (1996) investigated well-structured mathematical tasks and revealed that they promote thinking and reasoning, include multiple solution strategies and multiple representations of mathematics. Their study also revealed that well-structured mathematical tasksinclude memorization and use of algorithms with or without concept understanding, and promote complex thinking and reasoning. They contend further that well-structured mathematical tasks promote explanation, justification and conjecturing. Schoenfeld (2007) argued that well-structured mathematical tasks must address the following postulate: they should ensure students use of multiple solution strategies in their response to examination questions; ensure students produce mathematical explanations and justifications; and ensure students engage in high level thinking and reasoning. These characteristics are embedded intrinsically in the construct of mathematical proficiency which students might exhibit in their responses (Cragg and Gilimore, 2014; Greenes, 2014; Guberman and Leikin, 2013).

The purpose of this study was to investigate the types of strands of mathematical proficiency that learners displayed when responding to four topics namely: sequence and series, differential calculus, analytical geometry and trigonometry. The research question was:

What strands of mathematics proficiency can be identified in learner responses towards examination questions on sequences and series, differential calculus, analytical geometry and trigonometry?

2. Literature review

This research focused on the strands of mathematical proficiency that learners displayed in their response to the chosen final examination questions. Bartell, Webel, Bowel, and Dyson (2013) described mathematical proficiency as the advancement in mathematical skills and knowledge. Kilpatrick et al. (2001) defined mathematical proficiency as knowledge, expertise, competence and the facility of mathematics. Groves (2012) viewed mathematical proficiency as the ability to engage in mathematics with understanding, computing, applying, reasoning and engaging.

This study used four strands of mathematical proficiencies; procedural fluency, conceptual understanding, strategic competence and adaptive reasoning and according to Kilpatrick et al. (2001), the strands are interconnected, intertwined and interrelated.Procedural fluency describes thelearner’s ability of carrying out mathematics procedures in a manner that is flexible, accurate and efficient (Kilpatrick et al. 2001; Suh 2007). Procedural fluency also involve knowing when to use mathematical procedures, ‘knowing how’ of mathematical knowledge which is learner’s ability to quickly recall and accurately execute procedures (Zakaria and Zaini, 2009; Graven and Stott, 2012). Procedural fluency is often called computing which involves engaging in practice with procedures and manipulation of numbers, knowing ‘how to do it’ of mathematical knowledge.Furthermore procedural knowledge involves knowledge of operators and their conditions of application,sometimes referred to as automatized knowledge that occurs with minimal conscious attention that cannot be transformed to higher mental functions (Schneider and Stern, 2012; McCormick, 1997).

The second strand of mathematical proficiency, conceptual understanding refers to learners’ ability to comprehend mathematical procedures, concepts relations and operations (Kilpatrick et al. 2001; Groves, 2012; Rittle-Johnson and Alibali, 1999). Zakaria and Zaini (2009) argued that conceptual knowledge is mathematics that is rich in relationships, interconnections between ideas which explain and give meaning to mathematical procedures. Our observation here is that as conceptual understanding focuses on the ability to work with a variety of procedures simultaneously and with procedural fluency learners’ work with procedures in isolation. In their elaboration of conceptual knowledge, Schneider and Stern (2010) explained that conceptual understanding is knowing ‘why’ of mathematics knowledge whenconnecting related algorithms and knowledge items.McCormick (1997) argued that when learners exhibit the ability to link algorithms and knowledge items, conceptual knowledge is seen as general and abstract mathematical knowledge of the key principles and their interrelations in the mathematics domain.Schoenfeld (2007) described conceptual understandingas the explicit or implicit understanding of the principles that govern the mathematics domain coupled with the interrelations amongst parts of knowledge in the domain.

Zakaria and Zaini (2009) conducted a study on conceptual and procedural knowledge and findings of their study revealed that pre-service teachers were confident to solve problems that required routine procedures (exhibited procedural knowledge). They were unable to give reasons why they were using those procedures in certain questions (lack of conceptual knowledge). Our observation here is that this study shows that even teachers had a problem with conceptual knowledge which makes it highly possible that learners are not taught conceptually.Furthermore a study by Simpson and Zakaria (2004) that focused on applying mathematics in chemistry problemsshowed that there was no correlation in the attainment of procedural knowledge and conceptual knowledge. Through the analysis of textual responses of learners, the correlation seen in their findings showed that more learners were keen to apply procedural knowledge as opposed to conceptual knowledge(Simpson and Zakaria, 2004).

The third strand of mathematical proficiency, strategic competence, is learner's skills of formulating, representing and solving mathematical problems (Kilpatrick et al.2001). Strategic competence is sometimes called applying, which is the skill of using computations to solve routine and non-routine mathematical problemsthat requires learners to be fluent in procedural knowledge and some degree of conceptual knowledge (Groves, 2012). According to Schoenfeld (2007) strategic competence has two components of skills, the generative and evaluative skills which are also components of problem solving and that requires some degree of creativity and flexibility. From our perspective learners who had the ability to formulate, represent and solve mathematical problems without giving reasons 'why'those problems were solved in in a particular way, had procedural knowledge. Those who could give valid reasons of how they solved mathematical problems had conceptual knowledge. In our context this was in reference to questions that required derivation of a proof, construction and linking related and relevant or appropriate formulas as well as providing intrinsically the reasons behind the use of certain procedures in sequence and series, differential calculus, analytical geometry and trigonometry.

The last strand of mathematical proficiency used in this study is adaptive reasoning,refers to the learner's skills for logical thought, reflection, explanation and justification (Kilpatrick et al. 2001). It also involves the learner's capacity to think logically about relationship between mathematics concepts and relations (Kilpatrick el al. 2007; Suh, 2007). Schoenfeld (2007) stated that this strand of mathematical proficiency determined learner's capability of handling mathematics concepts, procedures and problem solving strategies. He contended that this was evident when learners justify the use of concepts, and procedures during problem solving and furthermore justify how problems are solved in a particular way. From the learners' responses to the questions we ascertained that adaptive reasoning included learner's ability to reason both formally and informally. Formal reasoning involved using the rules of logic and structures of proof (such as contradiction, induction and deduction), whilst informal reasoning included the utilization of semi-rigorous justifications and reasoning based on representations (Amir-Mofidi, Amiripour, and Bijan-zadeh, 2012). It implies therefore that learners with only procedural knowledge could mostly perform mathematical skills that required strategic competence but those with conceptual knowledge would exhibit both strategic competence and adaptive reasoning skills.

3. RESEARCH METHODS AND PROCEDURES

3.1The research design

This study is qualitative in nature. Qualitative research generates theory through interpreting evidence and data gathered from direct quotations and open ended questions (Volkwein, 2001).The data used in this study came from learner's responses to final examination questions which madethe study to be qualitative.

3.2 Data collection

The scripts were collected by the University of Johannesburg Faculty of Education project called Script Analysis. This study used 90 scripts from a batch of 1000 scripts, 45 from paper 1 and 45 from paper 2 which represented were used to generate theory. The scripts were purposefully selected, only those with substantial responses to the chosen questions were used. The scripts were in batches of 50 and we simply selected scripts in these range of marks, group A (GA) from 0-50, group B (GB) from 51-90 and group C (GC) from 90-200 marks. We used these categories only for the purpose of managing the learners' responses and hardly related the learners' achievements to the strands of mathematical proficiency they exhibited.

3.3 Data analysis

The data was analyzed using the instruments developed from theory on strands of mathematical proficiency with reference to learners' performance in the examination questions. This means that for both paper 1 and paper 2, 30 scripts were from learners who got from 0 and 50, group A (GA), 30 scripts were from learners who got from 51 to 90, group B (GB), 30 scripts were from learners who got from 90 to 200, group C (GC). This helped us to identify mathematical proficiencies that various ranges of achievement displayed in their response to the selected examination questions.

Cohen and Manion (1994) stipulated that the standard rules of content analysis included, the size of the data to be analyzed: is it a line, a paragraph or a phrase? What were the units of meaning? Are themes inclusive of all categories, or mutually exclusive? Data defined precisely with the mention of its properties, and lastly, was that data fitted in some category?Categories must be exhaustive. In the context of this study the size of the data were the learners' responses to examination questions that showed the spectrum of proficiencies that learners displayed. The categories were based on the meaning of mathematical proficiencies that were found in the review of literature. This provided the deductive analysis of the data because the categories were predetermined. The indicators for mathematical proficiencies were designed to address as much as possible proficiencies that learners displayed in their response to examination questions (see table 1).

Mathematical Proficiencies	**Performance By learners**	**Codes**
Conceptual Understanding	No response. It is either learners only wrote the question number or left a blank space.	CU0
	Learners display no knowledge of connecting procedures and rules.	CU1
	Learners connect mathematics procedures, concepts and algorithms with major errors.	CU2
	Learners connectmathematics procedures, concepts and algorithms with minor errors.	CU3
	Learners accurately connects and reconnects mathematical procedures, concepts and algorithms.	CU4
Procedural Fluency	No response. It is either the learner only wrote the question number or left a blank space.	PF0
	Learners display no knowledge of the procedure.	PF1
	Learnersshow insight of a procedure with major mathematical errors.	PF2
	Learners show acceptable knowledge of procedure with minor errors.	PF3
	Learners carry out mathematical procedures accurately and appropriately.	PF4
Strategic competence	No response to the question. It is either learners only wrote the question number or left a blank space.	SC0
	No knowledge of either formulating, representing or solving mathematical problems.	SC1
	Learners either formulate, represent or solve mathematical problems with some computational errors (no creativity).	SC2
	Learners formulate, represent and solve mathematical problems showing reasons of ‘how’ and minimal reasons	SC3

	'why' of mathematical knowledge (minimum creativity).	
	Learners formulate, represent and solve mathematical problems showing both the 'how' and 'why' of mathematical knowledge (creative response).	SC4
Adaptive Reasoning	No response to the question. It is either learners only wrote the question number or left a blank space.	AR0
	Learners display no skills for logical thought, reflection and justification.	AR1
	Learners use representations to display logical thought, reflection and justification.	AR2
	Learners use abstractions to display logical thought, reflection limited to mathematical reflection and explanation.	AR3
	Learners use abstractions to display logical thought with mathematical reflection, explanation and justification.	AR4

*Table 1: codes for mathematical proficiencies***.**

Forty five learner responses on the chosen questions were analyzed from the 2007 higher grade mathematics final examination and the strands of mathematical proficiencies were unveiled using codes as shown in table 1. We ascertain that it was challenging to analyze the strands in isolation, therefore we subscribed to Kilpatrick et al. (2001) view that they are intertwined and interconnected. Therefore in the table 2 we separated the strands into the two knowledge(s), procedural and conceptual. The skills, strategic competence and adaptive reasoning were captured as strands appearing in the knowledge(s). A question by question analysis as adopted from Luneta and Makonye (2010) was used to analyze learner's responses and the summary of the analysis is shown in table 2.

Examination Questions	**CATEGORIES OF MATHEMATICAL PROFICIENCIES**					
	Not Proficient		**Moderate Proficient**		**Proficient**	
	Procedural	Conceptual	Procedural	Conceptual	Procedural	Conceptual

Sequences and Series	PF0SC0 PF1SC1	PF0SC0CU0 PF1SC1CU1	PF3SC2 PF2SC1 PF2SC2	PF3SC2CU2 PF2SC1CU1 PF2SC2CU1	PF4SC4	PF4SC4CU4
TOTAL	143	69	33	11	47	10
Differential Calculus	PF0SC0 PF1SC1	PF0SC0CU0 PF1SC1CU1	PF3SC2 PF2SC1 PF2SC2	PF3SC2CU2 PF2SC1CU1 PF2SC2CU1	PF4SC4	PF4SC4CU4
TOTAL	195	126	31	52	44	92
Analytical geometry	PF0SC0 PF1SC1	PF0SC0CU0 PF0SC0CU0AR0 PF1SC1CU1 PF1SC1CU1AR1	PF3SC2 PF2SC1 PF2SC2	PF3SC2CU2 PF2SC1CU1 PF2SC2CU1	PF4SC4	PF4SC4CU4 PF4SC4CU4AR4
TOTAL	2	61	0	53	43	41
Trigonometry	PF0SC0 PF1SC1	PF0SC0CU0 PF0SC0CU0AR0 PF1SC1CU1 PF1SC1CU1AR0	PF3SC2 PF2SC1 PF2SC2	PF3SC2CU2 PF2SC1CU1 PF2SC2CU1 PF2SC2CU1AR1	PF4SC4	PF4SC4CU4 PF4SC4CU4AR2
TOTAL	44	286	38	155	33	144

Table 2: A summary of learner's responses to examination questions

Key: *PF0-4, codes for proficiency in procedural fluency; CU0-4, codes for proficiency in conceptual understanding; SC0-4, codes for proficiency in strategic competence; AR0-4, codes for proficiency in adaptive reasoning.*

Learners exhibited strands of mathematical proficiencies as shown in the table 2. In the analysis we categorized learners' proficiencies into strands that are shown in table 2. The analysis revealed three categories, not proficient, moderate proficient and proficient. The vignettes and a few questions below outline how the codes were generated from the learners' responses to the chosen examination questions.

In figure 1 learners responded to the following question:

5.1.2 Calculate the sum of all natural numbers less than or equal to 200, excluding those that are multiples of 7.

RECOMMENDED ANSWER IN MEMORANDUM

$S_{28} = \frac{n}{2}$ [2a + (n-1) d] (formula, learner to select from formula sheet)

$= \frac{28}{2}$[2(7) + (28-1)7] (substitution)

= 2842 (simplification, correct answer, sum of multiples of 7)

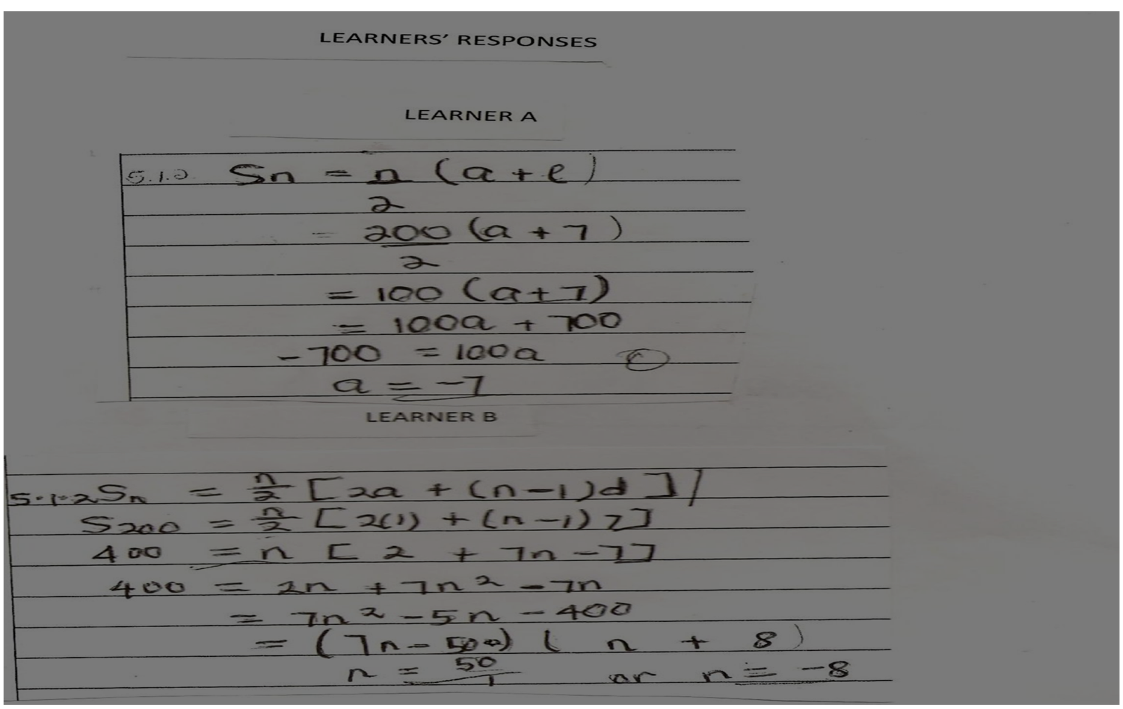
$S_{200} = \frac{200}{2}$ [2(1) + (200-1)1](substitution)

= 100[2 + 199]

= 20100 (simplification, correct answer, sum of all multiples equal or less than 200)

S_n = 20100 – 2842 = 17258 (difference, removing multiples of 7.

The vignette in figure 1 shows four examples of learner's work on this examination question on sequences and series.


LEARNERS' RESPONSES

LEARNER A

5.1.2 $S_n = \frac{n}{2}(a + l)$

$= \frac{200}{2}(a + 7)$

$= 100(a+7)$

$= 100a + 700$

$-700 = 100a$

$a = -7$

LEARNER B

5.1.2 $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{200} = \frac{n}{2}[2(1) + (n-1)7]$

$400 = n[2 + 7n - 7]$

$400 = 2n + 7n^2 - 7n$

$= 7n^2 - 5n - 400$

$= (7n - 50)(n + 8)$

$n = \frac{50}{7}$ or $n = -8$

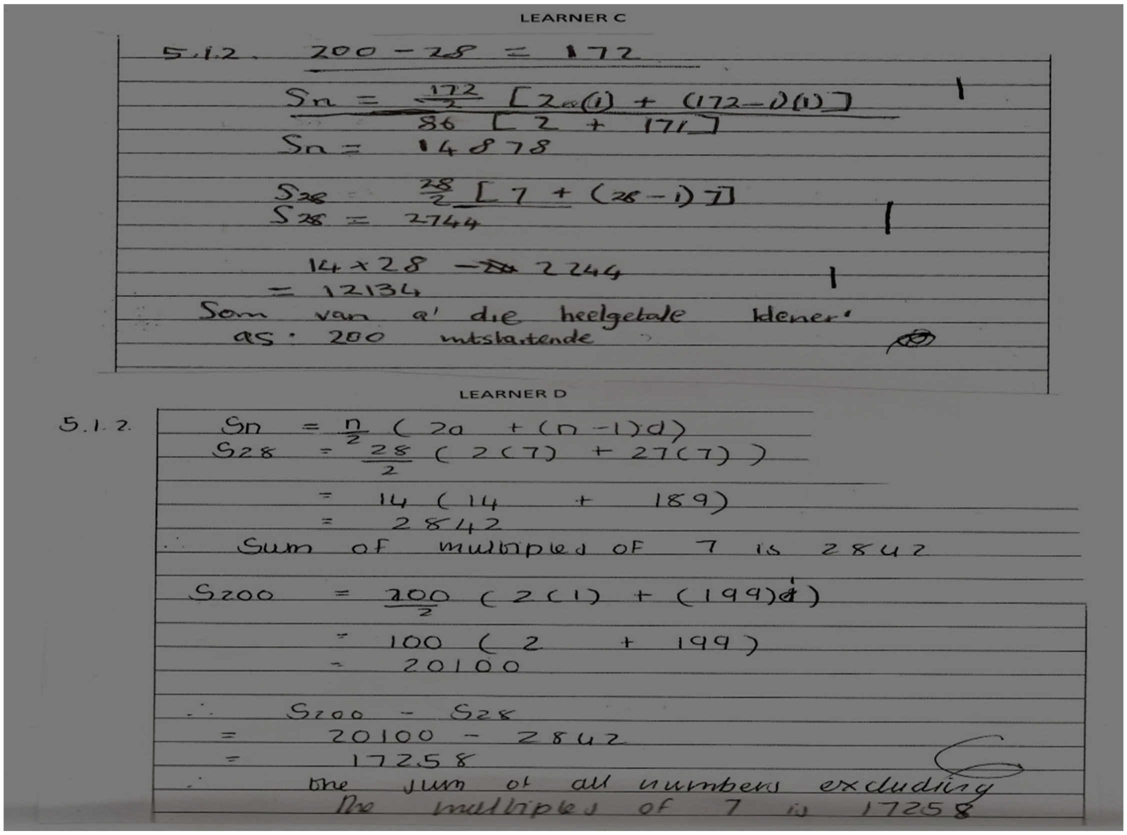


Figure 1: Learners' responses for question 5.1.2 on sequences and series.

Explanation of codes in figure 1

Learner A's response is one of the 10 who were not proficient on this question and were coded PF1 SC1. **PF1**; the learner chose the wrong formula from the formula sheet and the resultant solution was wrong. We classified this learner as one who had no knowledge of the procedure. **SC1**; Substitution and simplification were considered wrong because of the use of an incorrect formula.

Learner B was one of 12 that were coded PF2 SC2: PF2; Learner B chose the correct formula however incorrectly used the procedure of finding the sums. **SC2**; other values substituted correct however '200' is substituted incorrect and since follow up procedure is wrong then the resultant substitution is wrong.

Learner C was one of 2 that were coded PF3 SC2**: PF3**; learner first find the correct difference 'subtracting 28 from 200' , chose correct formula, substituted correct however simplification was inconsistent with the procedure of finding the sum. **SC2**; Substitution in the formula was correct, followed by wrong simplification.

Learner D is one example of 4 of those that were coded PF4 SC4: **PF4**; the learner chose a correct formula and used procedure appropriately to find the sum. **SC4**; learner D substituted correct in the formula and simplified correct for both sums (S_{28} and S_{200})then calculated the difference between sums accurately to reach the correct answer.

Question 6.2.2 (paper 1) on the introduction of differential calculus is shown.

6.2 Determine $\frac{d}{dx}$ in each of the following:

6.2.2 y (x + 1) = $3x^2 + 4x + 1$

Suggested answer in memorandum

6.2.2 y (x + 1) = $3x^2 + 4x + 1$

y (x + 1) = (3x + 1)(x + 1) (finding factors)

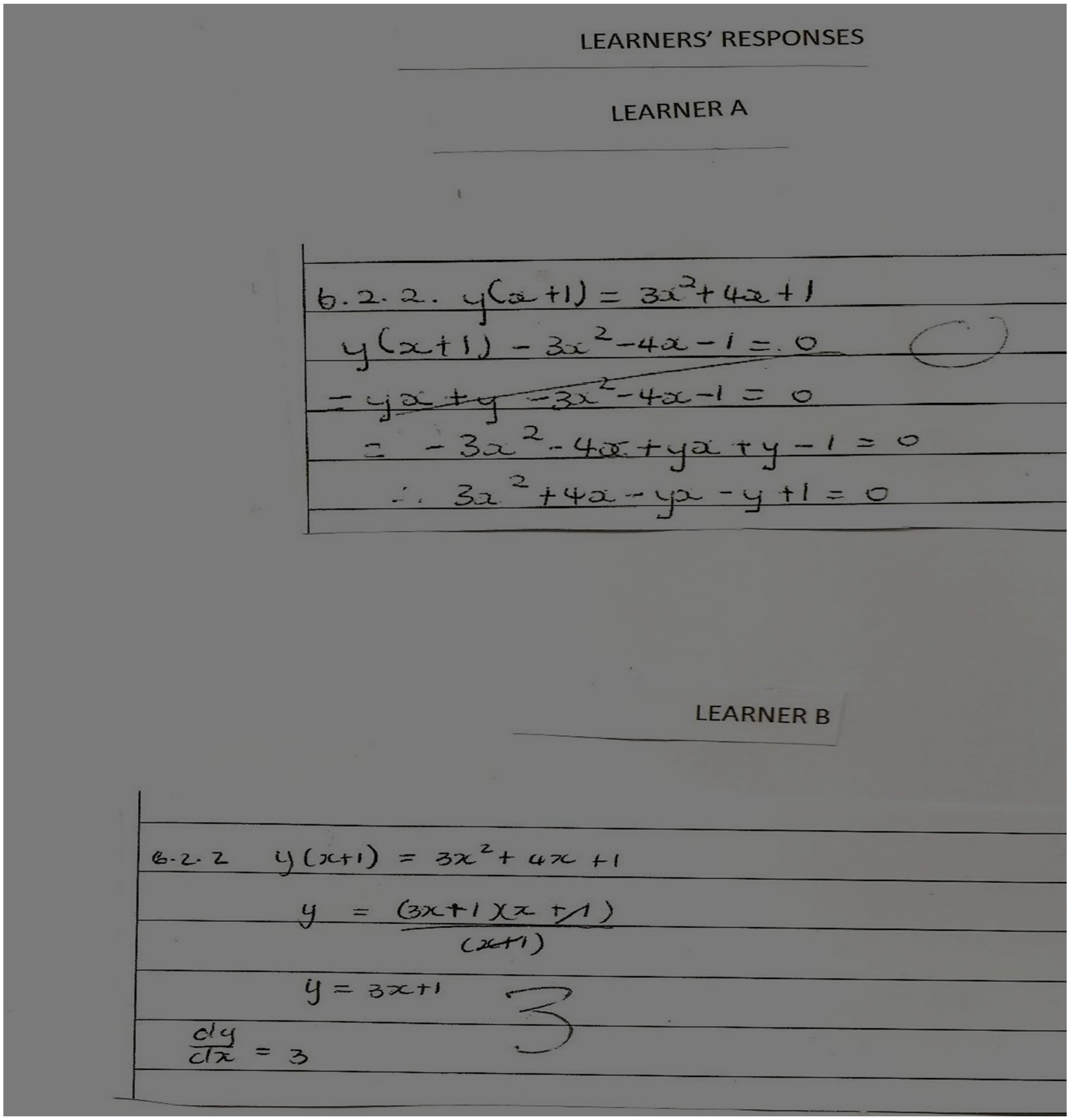
y = 3x + 1 (simplification)

$\frac{dy}{dx}$ = 3 (finding the derivative)

Explanation of codes in figure 2

The vignette on figure 2 shows two examples of learner's responses for examination questions on differential calculus that demanded conceptual knowledge. Learner A's response was one of the 17 classified as "not proficient" and was one of those coded PF1 SC1 CU1. **PF1**; Procedure is incorrect**.** **SC1**; simplification is wrong and learner did not find the derivative**.** **CU1**; learner did connect procedures, factorization and finding derivative.

Learner B's response was one of the 10 classified proficient, and thus coded PF4 SC4 CU4. **PF4**; procedure of finding the derivative was correct. **SC4**; factorization and calculating the derivative is correct. **CU4**; learner comprehended procedures correct, (factorization and finding derivative).


LEARNERS' RESPONSES

LEARNER A

$6.2.2.\ y(x+1) = 3x^2+4x+1$

$y(x+1) - 3x^2 - 4x - 1 = 0$

$= yx + y - 3x^2 - 4x - 1 = 0$

$= -3x^2 - 4x + yx + y - 1 = 0$

$\therefore 3x^2 + 4x - yx - y + 1 = 0$

LEARNER B

$6.2.2\quad y(x+1) = 3x^2 + 4x + 1$

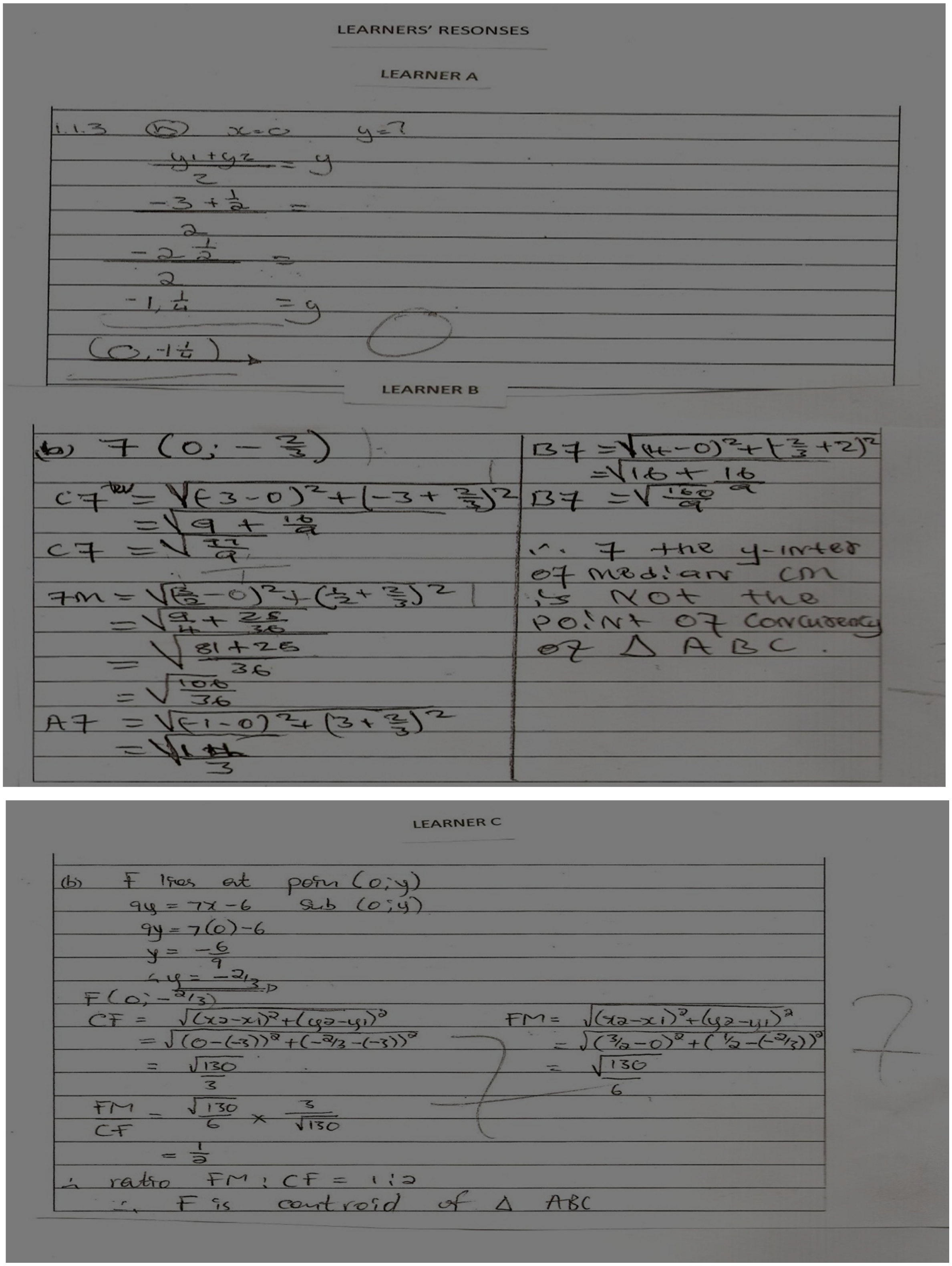
$y = \dfrac{(3x+1)(x+\cancel{1})}{(x+1)}$

$y = 3x+1$ 3

$\dfrac{dy}{dx} = 3$

*Figure 2: Learners' responses for question 6.2.2 on differential calculus***.**

Thevignettes on figure 3 are examples of learners' responses on analytical geometry.



LEARNERS' RESONSES

LEARNER A

1.1.3 (b) $x = 0$ $y = ?$

$\frac{y_1 + y_2}{2} = y$

$\frac{-3 + \frac{1}{2}}{2} =$

$\frac{-2\frac{1}{2}}{2} =$

$-1\frac{1}{4} = y$

$(0, -1\frac{1}{4})$

LEARNER B

(b) $F(0; -\frac{2}{3})$

$CF = \sqrt{(-3-0)^2 + (-3+\frac{2}{3})^2}$

$= \sqrt{9 + \frac{16}{9}}$

$CF = \sqrt{\frac{97}{9}}$

$FM = \sqrt{(\frac{3}{2}-0)^2 + (\frac{1}{2}+\frac{2}{3})^2}$

$= \sqrt{\frac{9}{4} + \frac{25}{36}}$

$= \sqrt{\frac{81+25}{36}}$

$= \sqrt{\frac{106}{36}}$

$AF = \sqrt{(-1-0)^2 + (3+\frac{2}{3})^2}$

$= \sqrt{\frac{1\ [illegible]}{3}}$

$BF = \sqrt{(4-0)^2 + (\frac{2}{3}+2)^2}$

$= \sqrt{16 + \frac{16}{9}}$

$BF = \sqrt{\frac{160}{9}}$

∴ F the y-inter of median CM is NOT the point of concurrency of ΔABC.

LEARNER C

(b) F lies at point $(0; y)$

$9y = 7x - 6$ sub $(0; y)$

$9y = 7(0) - 6$

$y = \frac{-6}{9}$

$y = -\frac{2}{3}$

$F(0; -\frac{2}{3})$

$CF = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(0-(-3))^2 + (-\frac{2}{3} - (-3))^2}$

$= \frac{\sqrt{130}}{3}$

$FM = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(\frac{3}{2}-0)^2 + (\frac{1}{2} - (-\frac{2}{3}))^2}$

$= \frac{\sqrt{130}}{6}$

$\frac{FM}{CF} = \frac{\sqrt{130}}{6} \times \frac{3}{\sqrt{130}}$

$= \frac{1}{2}$

∴ ratio $FM : CF = 1 : 2$

∴ F is centroid of ΔABC

Figure 3: Learners' responses for question 1.1.3.b on analytical geometry.

For question 1.1 3 (paper 2) learners responded to the following question.

In the diagram alongside, A(-1; 3), B(4, -2) and y

C (-3; k) are three points in theA (-1; 3)

Cartesian plane D

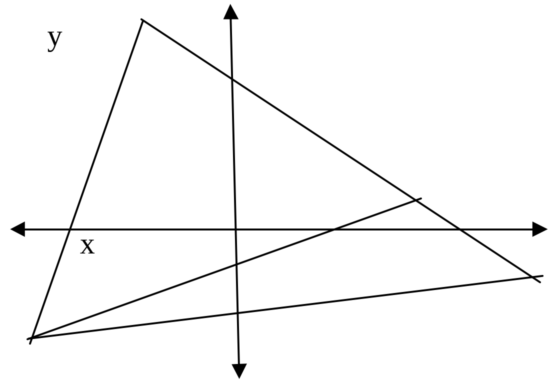
AB = BC

CM is the median with M on ABM

D is the y-intercept of straight line AB. F

B(4;-2)

C (-3;k)



1.1.3 Hence if k = -3, determine:

(b) Whether F, the y-intercept of the median CM, is the point of concurrency of the medians of triangle ABC.

Suggested answer in memorandum

As x = 0 then y =$\frac{-2}{3}$ (the point is on the y-axis and substituting with zero)

F (0,$\frac{-2}{3}$) (co-ordinates of F)

Mid-point of AC 'N'

N = [$\frac{-1-3}{2}$; $\frac{3-3}{2}$] (substitution in the formula for mid-point)

N = (-2; 0) (co-ordinates of mid-point)

Mab = $\frac{0+2}{-2-4}$ (calculating gradient of line AB

= $\frac{-1}{3}$ (answer, the gradient of AB)

y = $\frac{-1}{3}(x + 2)$ (substituting in the general formula for straight line)

y = $\frac{-1}{3}x - \frac{2}{3}$ (resultant equation)

As x = 0 then y = $\frac{-2}{3}$ (x-intercept and substituting with zero)

Then F is the point concurrent (it is where the mid points meet)

Explanation of codes in figure 3

Learner A's response was one of 5 which were classified as "not proficient" and coded PF1 SC1 CU1 AR1:**PF1**; procedure not known by this learner.**SC1**; learner cannot follow any problem solving

procedures to solve the problem.**CU1**; learner cannot connect procedure (gradient, x-intercept and mid-point) to prove concurrency.**AR1**; Learner had no skills of proving for concurrency.

Learner B was one of 11 responses for learners who were classified moderate proficient and coded PF2 SC2 CU2 AR2**: PF2**; learner chose correct procedure, distance formula, to prove concurrency however the use of the procedure had errors that did not lead to the required conclusion.**SC2**;learner substituted correct in the distance formula however the simplification was wrong and did not lead to the required conclusion.**CU2**; learner comprehended two distances to prove concurrency which eventually did not lead to the required conclusion.**AR2**; Learner used two distances to prove median, an acceptable proof howeverwere made errors that did not warrant an acceptable proof.

Learner C's response was one of the 11 classified as proficient and coded PF4 SC4 CU4.**PF4**; Learner used correct procedure, distance formulae to correctly prove concurrency.**SC4**; learner substituted correct in the formulae and the simplification led to the correct conclusion of proving concurrency.**CU4**; learner comprehended two distances correct to prove concurrency.

The last vignette on figure 4 was a learners' response to trigonometry.

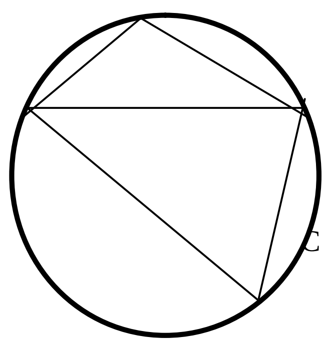
For question 6.1.1 (paper 2) learners responded to the question below.

In the diagram alongside, BD is a chord of circle ABCD.

BD = BC = p

A

B pp D D



C

6.1.1 Show that CD = -2p.cos A

Suggested answer in memorandum

BD = BC = p (given in the question)

< C = <D_2 (base angles of an isosceles)

<C + <A = 180^o (angles in a cyclic quad sum)

<C = 180^o **-** <A (making angle C the subject of the formula)

$BD^2 = BC^2 + DC^2 – 2(BC). (DC).Cos (180^o – A)$ (cosine rule)

$p^2= p^2 + DC^2 - 2p (DC) (-Cos A)$ (substitution)

$0 = DC^2 + 2p (DC) Cos A$ (simplification)

$-DC^2 = -2p (DC) Cos A$ (simplification)

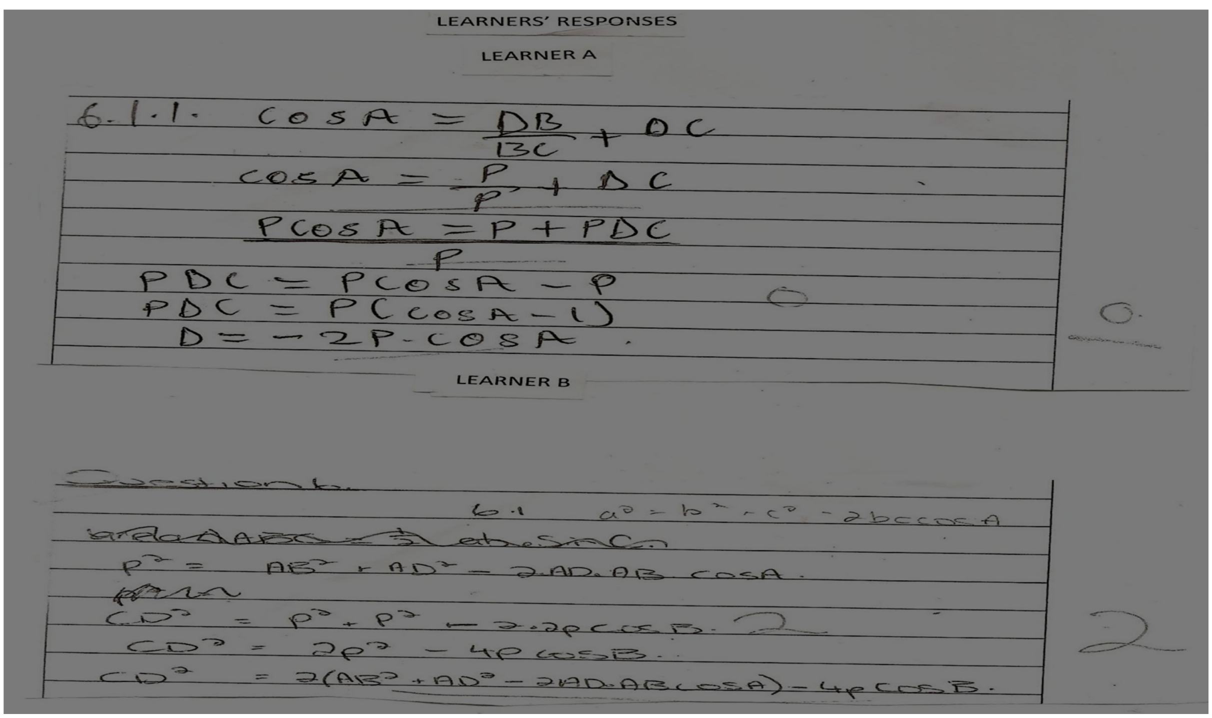
DC = **-**2p Cos A (answer, proved)

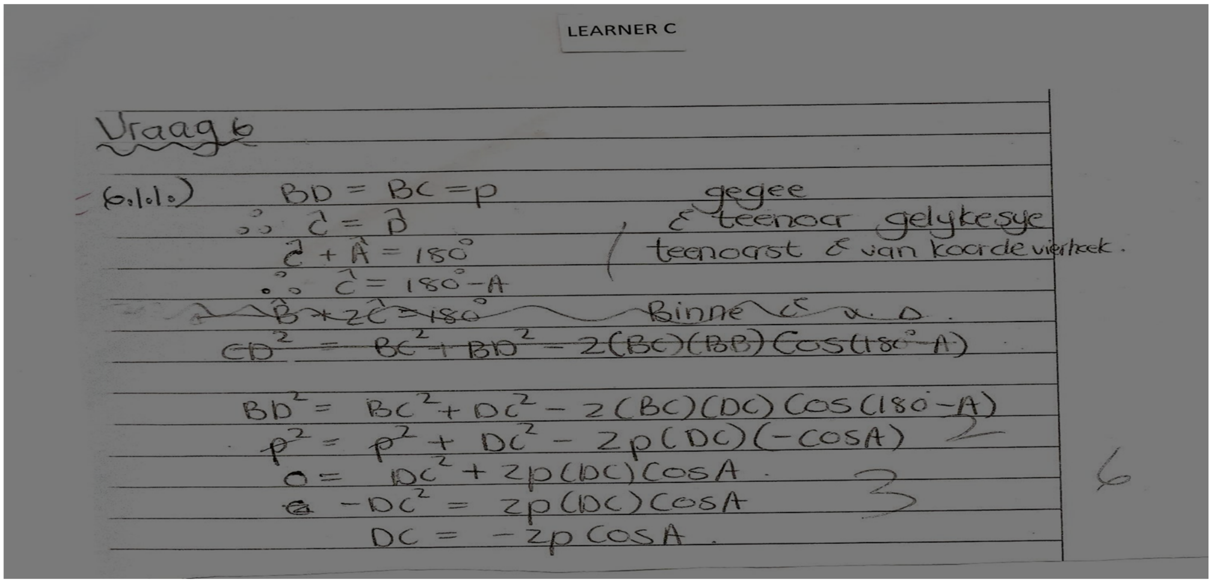
Explanation of codes in figure 4

The vignette on figure 4 showed three learners' responses to a question on trigonometry that demanded conceptual knowledge.Learner A's response was one of those thirteen classified as "not proficient" and coded PF1 SC1 CU1:**PF1**; Learner chose wrong procedure resulting in the solution being mathematically wrong.**SC1**; All substitutions and simplifications considered wrong due to wrong procedure used.**CU1**; connection of procedures considered wrong due to the use of wrong procedure.

Learner B's response was one of the twelve classified as "moderate proficient" and was coded PF2 SC2 CU1:**PF2**;the learner chose the correct procedure however execution of the procedure the procedure was wrong. **SC2**;learner substitutes correct but simplification is wrong.**CU1**; learner Bdid not show that procedures need to be connected to solve the problem. (Cosine rule and angles of a cyclic quad).

Learner C's response was one of the two classified as proficient and coded PF4 SC4 CU4:**PF4**;the learner used correct procedure to get to the required answer. **SC4**; learner C substituted and simplified correct which lead to the correct answer.**CU4**;the learner comprehended procedures correct (cosine and angles of a cyclic quad).




LEARNER C

Vraag 6

6.1.1) $BD = BC = p$ gegee

$\therefore \hat{C} = \hat{D}$ ∠ teenoor gelyke sye

$\hat{C} + \hat{A} = 180°$ teenoorst ∠ van koordevierhoek.

$\therefore \hat{C} = 180° - A$

$\cancel{\hat{B} + 2\hat{C} = 180°}$ Binne ∠ v. Δ.

$\cancel{CD^2 = BC^2 + BD^2 - 2(BC)(BB)\cos(180° - A)}$

$BD^2 = BC^2 + DC^2 - 2(BC)(DC)\cos(180° - A)$

$p^2 = p^2 + DC^2 - 2p(DC)(-\cos A)$ 2

$0 = DC^2 + 2p(DC)\cos A$

$-DC^2 = 2p(DC)\cos A$ 3

$DC = -2p\cos A$

6

Figure 4: Learners' responses for question 6.1.1 on trigonometry

4. RESULTS AND DISCUSSION

The analysis of the learners' scripts revealed that learners' proficiencies ranged from "not proficient" to "moderate proficient" and "proficient" as shown in the diagram below:

|---► |---► |

Not Proficient moderate proficient Proficient

Figure 5: mathematical proficiencies continuum found in learners' scripts.

We viewed the strands of mathematical proficiencies as discursive learners' responses that were located between three place holders shown in figure 5. The strands of mathematical proficiency shown in table 2 showed the journey learners embark on their quest to become proficient in mathematics.

There were three categories of mathematical proficiencies that were displayed by learners in their response to examination questions on all four topics analyzed for questions that demanded either procedural or conceptual knowledge. For learners who were *not proficient,* the strands showed learners who left blank spaces and those that used wrong mathematical procedures resulting in their responses being mathematically wrong. For the category *moderate proficient*, there were a variety of indicators that showed this category; some learners only wrote the correct formula, some wrote the correct formula, substituted correct and simplified wrong, some chose correct formula, substituted correct, simplified correct and wrote wrong conclusion, some chose correct formula, failed to comprehend procedures, some comprehended correct and failed to justify their answers. For learners who were *proficient,* these learners used correct procedures, comprehended procedures correctly, substituted correct, simplification was correct and reached the correct answers or conclusions.

For sequences and series, there were learners who were *not proficient, moderate proficient* and *proficient*. There were 63.13% of learners who were *not proficient* on examination questions that demanded procedural knowledge. 76.67% of learners were not proficient on examination questions that demanded conceptual knowledge. The findings were that these learners left blank spaces and others used wrong mathematical procedures resulting in wrong answers. About 14.78% of the learners were *moderate proficient* on questions that demanded procedural knowledge and 12.22% of the learners that responded to the examination questions that demanded conceptual knowledge were *moderate proficient.* The study showed only 21.08% learners that responded to examination questions that demanded procedural knowledge were proficient and also that only 11.11% learners that responded to the examination questions that demand conceptual knowledge were *proficient.*

On the questions on differential calculus, the same categories were used, *not proficient*, moderateand *proficient.* The study revealed that 72.22% of the learners that wrote the examination questions that demanded procedural knowledge in differential calculus were *not proficient* and 46.67% of the learners that responded to questions that demanded conceptual knowledge were also *not proficient.* There were 11.48% of the learners that responded to questions that demanded procedural knowledge that were *moderate proficient* and 19.26% of the learners that responded to questions that demanded conceptual knowledge were also *moderate proficient.* Lastly, there were only 16.3% of the learners that responded to questions that demanded procedural knowledge that were *proficient* and 34.07% of the learners were proficient on questions that demanded conceptual knowledge.

On analytical geometry, 4.44% of the learners responded to questions that demanded procedural were *not proficient* and 39.35% of the learners that responded to the questions that demanded conceptual knowledge were also *not proficient.* No learners were *moderate proficient* on questions that demanded procedural knowledge and 34.19% of the learners that responded to questions that demanded conceptual knowledge were *moderate proficient.* Lastly, 95.56% of the learners that responded to examination questions that demanded procedural knowledge were *proficient* and 26.45% of the learners that responded to the examination questions that demanded conceptual knowledge were *proficient* on this topic. This implied that majority of the students had acquired procedural knowledge of solving analytical geometry questions in the examination.

For trigonometry 57.04% of the learners that responded to questions that demanded procedural knowledge were *not proficient* and 48.89% of the learners that responded to questions that demanded conceptual knowledge were also *not proficient*. 26.76% of the learners that responded to questions that demanded procedural knowledge were *moderate proficient* and 26.3% of the learners that responded to questions that demanded conceptual knowledge were also *moderate proficient*. There were also 23.34% of the learners that responded to questions that demanded procedural knowledge that were *proficient* and 24.62% of the learners were also *proficient* on questions that demanded conceptual knowledge.

5. CONCLUSION

From this study it is safe to say that it was most likely that a majority of learners in South Africa left high school not proficient in sequences and series for examination questions that demanded both procedural and conceptual knowledge. Pointers may be directed to teaching procedurally by teachers who themselves struggled with conceptual knowledge as per the findings of the study by Zakaria and Zaini (2009). The study also revealed that there were learners who left high school moderate proficient in examination questions on sequences and series. It was also most likely that a low number of learners left high school being proficient in examination questions on sequences and series that demanded procedural knowledge or conceptual knowledge. This increased the possibility that most of learners leaving high school struggled in tertiary courses that demanded the knowledge of sequences and series such as engineering.

The study has also revealed that it was most likely that a majority of learners left high school not proficient in examination questions on differential calculus that demanded procedural knowledge and this is contrary to studies on procedural knowledge such as Bartell et al. (2013) which showed that learners are most likely to exhibit procedural knowledge. It was most likely that half learners left high school not proficient on examination questions on differential calculus, a prerequisite for studies in engineering and other natural science careers. Most of the learners exhibited basic algebra error concurring with the study byLuneta and Makonye (2010).

Furthermore the study revealed that it is more likely that more learners exhibit proficiency on examination questions on analytical geometry that demanded procedural knowledge. It was alsoobserved that a substantial number of learners who left high school were moderate proficient in examination questions on analytical geometry that demanded both procedural and conceptual knowledge. Half the numberof the learners from the analysis were not proficient in examination questions on trigonometry that demanded both conceptual and procedural knowledge. There was also a considerable number of learners who left high school that were moderate proficient in examination questions on trigonometry that demanded both procedural and conceptual knowledge. There was low number of learners that left high school proficient in examination questions on trigonometry that demanded both procedural and conceptual knowledge. Our overall observation from this analysis was that most learners were more proficient in analytical geometry than the other chosen examination questions.

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