International Journal of Education Sciences (IJES)
Received on 26. 08. 2014/01. 10. 2014
REF. NO. IJES-526
(Always refer our Reference No. for all correspondence)
Exploring The Stages of Polya's Problem Solving Model During Collaborative Learning: A Case of Fractions
Deonarain Brijlall
Department of Mathematics, Durban University of Technology, South Africa
E-mail: deonarainb@dut.ac.za

Exploring The Stages of Polya's Problem Solving Model During Collaborative Learning: A Case of Fractions

Deonarain Brijlall

Department of Mathematics, Durban University of Technology, South Africa
E-mail: deonarainb@dut.ac.za

KEYWORDS Collaborative learning, problem-solving, Polya model, fractions

ABSTRACT This article reports on a small-scale action research which investigated the processes involved in problem solving in a mathematics class. Grade ten learners ($n = 47$) at a junior secondary school in Kwa-Zulu Natal, South Africa were involved in the study. The participants from two classes attempted the solution of tasks involving the fraction concept. In one class learners worked in groups and in the other class learners worked individually. The two tasks involved solving mathematics word problems in the light of the present outcomes based education dictates. A qualitative method was adopted in data capture and analysis. Social constructivism was adopted as a theoretical framework and the stages advocated by Polya were interrogated when analysing the learner responses on their problem solutions. The results revealed that those learners working in groups demonstrated most of the stages of the Polya linear problem solving model. The findings helped identify which stages of the model promoted effective problem solving and make recommendations to mathematics classroom practitioners who engage their learners in problem solving.

TRANSFORMATION IN EDUCATION CURRICULUM

Many studies (Maharajh, Brijlall & Govender, 2008; Schoenfeld, 1992) have investigated the successes of collaborative learning. These studies point to improved higher order learning abilities as a consequence of collaborative learning. The question not frequently explored is what mental mechanisms are being put in place to enforce such improved learning abilities. Hence this study delved on identifying the rationale for these improved learning skills by reflecting on a problem solving model introduced by mathematician George Polya. This study was conducted within an Outcome Based Education (OBE) paradigm. Outcome based education presently forms the foundation of the current curriculum in South Africa. Despite an envisaged revamp of OBE certain principles still remain. For example, that it is encouraged that a learner-centered and activity based approach to education be emphasised for the learners to learn effectively in mathematics (DoBE, 2010), will remain as a principle. Learners should be able to reflect on and explore a variety of strategies. They should also be culturally and aesthetically sensitive across a range of social contexts. However, studies like Tobias (2006) have provided evidence to indicate that students demonstrate a sense of being mathematically helpless when it comes to word problems. Most of such investigations allowed learners to solve problems on an individual basis. This study hence decided to look at collaborative learning. Other studies like Brijlall (2008; 2011) and Maharajh, Brijlall & Govender (2008) explored collaborative learning in multilingual, high school classrooms and tertiary students, respectively. Teachers' views on practical work, whilst working with fractions, were investigated by Maharaj, Brijlall & Molebale (2007). The findings in those studies revealed that collaborative learning benefitted mathematical learning. However, studies involving grade ten learners were not carried out by these researchers and this paper reports on such an investigation. Furthermore, this paper explores a possible relationship between successful problem solving and the levels of Polya's model.

The learning of Mathematics can be fun and enjoyable if taught in a familiar context related to the culture of the learners. Difficulty in problem solving in mathematics arises when a context is unfamiliar to them (Brijlall, 2011). They should therefore be provided with problem tasks which have given information familiar to their life experiences. For this reason the two tasks provided comprised information the learners could relate to. Also, the problem solving skills should be entrenched in the minds of the learners for use in later years. It was shown by Loji (2012) that higher order thinking needs to be developed early to enhance problem solving skills and discovery of new knowledge.

CONSTRUCTIVISM

According to constructivist Glasersfeld (1987) constructivism is seen as a cognitive theory of learning. It is emphasized that knowing is thought of as knowledge constructed by the learner as a process of adaptation based on the learners experience and constantly modified by the learner's experience. Thus in constructivism the learner does not discover an independent, pre-existing world outside his/her mind. There seems to be an implication that a learner cannot know, say a certain Mathematical concept unless it is done through their experience, and the learner can only know what they have constructed and modified according to their further experience. According to this theory, knowledge is viewed as fitting with experience, so that if the experience changes the knowledge also get modified. It is therefore important that the learners are led to talk about their thoughts to each other and to the teacher. To talk about what one is doing, ensures that one is examining it. In this study the result of such examinations led the learners to discuss their view of the problem and their own tentative approaches.

A constructivist sees the individual as trying to make sense of his/her experiences with the intention that the learners construct their own meaningful methods. Thus much of cognition is problem solving while little of what typically occur in school classroom could be considered problem solving because the learner is rarely allowed to make decisions (Grayson, 1994). In the constructivist approach, learners are assumed to construct their own mathematical conceptual understanding as they take part in cultural practices, and whilst they interact with each other. Inherent in this cultural perspective is the notion that students must have a point of view that exists as a central aspect of knowledge (Schoenfeld, 1992, p.340). According to the constructivists, as teachers we also need to reflect on the developmental progression of thought processes thinking to understand the wide range of thinking patterns of students in class and to plan task for groups and individuals. In so doing we move beyond traditional teaching and become curriculum builders (Steffe, 1991).This means that both teachers and learners become more responsible for learning. It is therefore imperative that mathematics teachers adapt learning design to focus on better learner understanding. We hence, decided to approach the problem solving, by allowing learners to interact with each other and communicate their ideas to encourage the solution of the given problems. Within groups, peer engagement necessitated a social situated for collective decision making. The two problem tasks chosen were based on knowledge of fractions and fraction manipulations. This topic was chosen as Maharaj et al (2007) found that most learners struggle with fractions in the middle school mathematics.

COLLABORATIVE LEARNING

We are constantly experimenting with ways we can employ to achieve success in the teaching and learning of Mathematics. The teacher plays a pivotal role in this regard. For instance, Ramnarain (2011) showed that the teachers used questioning to support learning in Science. We suggest that learners can derive benefit from working with peers. Barkley et al. (2005, p.4) defined collaborative as to work with another or others. This means that students are working in groups to achieve shared learning goals. They also called this learning as cooperative learning, team learning, group learning, or peer-assisted learning. Cooperative learning requires students to work together on a common task, sharing Information and supporting one another. This project formulated the following research question based on the principles fostered in outcomes based education (OBE) and collaborative learning: *How do learners engage the processes postulated by Polya when solving mathematics word problems during collaborative learning*? In pursuing this question we emphasise that we did not intend comparing collaborative learning with individual learning but to explore the prevalence of the stages of Polya's linear problem solving model (PLM) in each case.

Knowledge acquiring perception of the learner

Constructivists indicate that knowledge is formed as a learner acts upon a problem. Stears & Gopal (2010) reminded us that the knowledge children bring to the classroom may find expression in a variety of activities. The activity in this project involved the solving of word problems. The two tasks were non-routine problems because learners had not seen them during the teaching hours, but were supposed to be solved using their previous experience or their pre-existing knowledge relating them to their real-life situations. Since the school was located in a rural area, learners were familiar with the animals that they were given to work with, even Task 2 dealt with familiar concepts of fractions which were dealt with earlier in class. The following word problems were presented to learners to solve:

Task 1 On a farm I have goats and chickens. My son counted 70 heads and my daughter counted 200 legs. How many chickens and goats do I have?
Task 2 A research surveying marriages involved adult couples above the ages of 18. The fraction of men in this

population who are married is 3/5. What fraction of the population is single women?

The abstractions made are constructed by the learner and the learners discover the properties of the solution from the action. This discovery is a construction by a learner from what he or she already knows. In Mathematics this becomes more evident because learners are just introduced to a problem like the one of calculating the number of goats and chicken. How they solve this problem relies on their pre-knowledge and previous experiences in problem solving in mathematics.

We observed that the students were very interested and involved in the activity. The high involvement of the student is illustrated by the fact that some of them having finished Task 1 started by them trying some other possibilities such as drawing 70 heads. They started to put the required legs on the diagram used. One of the aims of teaching problem solving is for learners to communicate using their own meaningful methods, (Glasersfeld 1987). The grade ten syllabus also requires learners to identify, solve problems and make decisions using critical and creative thinking. Grade ten learners are required to work effectively with others as members of a team, group organization and community, (DOE; 2003). We explored whether the stages of Polya (1975) were implemented. Polya suggested that problem solving should follow the following stages: (a) understanding the problem, (b) devising a plan, (c) carrying out the plan and d) looking back. Each level (which Polya called principle) is summarized below: 1) First stage of PLM: *Understanding the problem* - this seems so obvious that it is often not even mentioned, yet learners are often hindered in their efforts to solve problems simply because they do not understand it fully or even in part, 2) Second stage of PLM: *Devising a plan* - he mentioned that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems, 3) Third stage of PLM: *Carrying out the plan* - this step is usually easier than devising a plan. In general, all you need is care and patience, given that you have the necessary skills. Persist with the plan that you have chosen. If it continues not to work discard it and choose another. We should not be misled, this is how mathematics is done, even by professionals and 4) Fourth stage of PLM: *Looking back* - he mentioned that much can be gained by taking the time to react and look back at what you have done, what worked, and what did not. Doing this will enable you to predict what strategy to use to solve future problems.

RESEARCH METHODOLOGY

A qualitative research method was adopted and focused on the teacher as a researcher. This focus of enquiry enhanced the action research carried out in this study. The qualitative paradigm was employed as the investigation was not concerned with statistical accuracy, but with detailed and in-depth analysis. The main focus of the study was to explore the learners' experiences during the mathematics class in the problem solving tasks. The data which was captured to answer the research question was suitable for a qualitative treatment. The main participants in the study were 24 grade ten learners. This is a sample out of 47 learners in grade ten classroom. The 24 learners consisted of 12 boys and 12 girls whose ages range from 14 to 17 years. The 24 learners were divided into 6 groups of 4 and 23 were working individually. Semi structured questionnaires were used to establish what transpired in and out of the problem solving class during the research period by grade ten learners. Learner conversations were audio recorded. The researcher reflected classroom activity and deciding on what could inform future action. Discussion were held with the learners by the observer and the responses of learners, expressions of the views on what was observed, and how these reflections could inform pedagogy were recorded.

RESULTS AND ANALYSIS

Comparison in achievement between the two classes

For the sake of the interview twelve learners were chosen, six from each class. We decided to choose the individual learners with about the same academic achievement as the group leaders. In this way we assumed that the contribution made by the group learners to the written responses will be similar to the contribution made by the individual learners. Their March term score for Mathematics were used. Table 1 shows the match of scores.

Table 1: Scores for March test

Group leader (%)	Individual (%)
A - 73	**1 - 74**
B - 68	**2 - 68**
C - 63	**3 - 60**
D - 51	**4 - 48**
E - 40	**5 - 39**

F - 34	**6 - 36**

Learners' general information

We recorded general information including gender and language of the two groups of learners. Hossain & Tarmizi (2012) explored gender-related effects of group learning in mathematics. Despite the fact that we were not focussing on gender issues in this study we thought it interesting to note such details to better understand the background of the participants.. All participants were second language English speaking. Table 2 and Table 3 show this information.

Table 2: Group information

Groups	A	B	C	D	E	F
Age	14	15	15	16	16	**16**
Gender	F	F	F	M	F	**F**
Home Language	**IsiZulu**	**IsiZulu**	**IsiXhosa**	**IsiZulu**	**IsiZulu**	**IsiZulu**

Table 3: Individuals information

Individual	1	2	3	4	5	6
Age	16	16	17	14	15	**16**
Gender	F	M	M	F	F	**F**
Home Language	**IsiZulu**	**IsiZulu**	**IsiZulu**	**IsiZulu**	**IsiZulu**	**IsiZulu**

The teaching of problem solving in grade 10 class

Participants were asked to construct their own meaningful methods to solve problems. In order to check on these steps we proceeded with the analysis of their solutions by focusing on the following questions: (1) Which stages of PLM are demonstrated in the learners written solutions? and (2) How was the mathematical correctness of the solutions aided by these stages of PLM?

Instructions were specified by the researcher to the learner when they commenced the task. They were asked to thoroughly read the information given in each question and answer both tasks.

During this time the researcher walked around observing learner engagement. The researcher found that learners were actively involved in the activity, they were communicating with each other in the groups, debating about their solutions until they reached the solution. While they were busy sharing ideas and information the group leader was jotting down what was supposed to be presented to the class.

Learner's responses

The written responses of the twelve learners were characterized by the following codes:

a. **Correct responses –** Correct mathematical ideas used and got task correct.

b. **Incorrect responses –** Task done but nothing correct.

c. **Completely cannot respond**- No work done but the blank answer sheet submitted.

d. **Partially correct responses**- Some steps done correctly.

The group responses are summarized in Table 4.

Table 4: Group responses

	Correct responses		**Incorrect responses**		**Completely cannot respond**		**Partially correct**	
	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	**Task 2**
	A	A		D			C	
	B	B					F	**F**
	D	C		E			E	
Total	**3**	**3**	**0**	**2**	**0**	**0**	**3**	**1**

The effect of collaborative learning seemed to impact positively in problem solving. This we deduce since: 1) fifty percent of each task was successfully solved, 2) all groups provided a response, 3) only seventeen percent proved unsuccessful and 4) twenty five percent of the learners could provide partial responses which could mean that the mathematical learning process is taking place.

Group A responses were correct for both tasks. The written responses possessed features that indicated all four stages of PLM. In their responses it was clear that they understood the problems, devised a plan, carried out the plan and lastly they looked back. This group first wrote: "*If there are*

70 heads of goats and chicken and 200 legs" at the beginning. They then drew diagrams as follows:

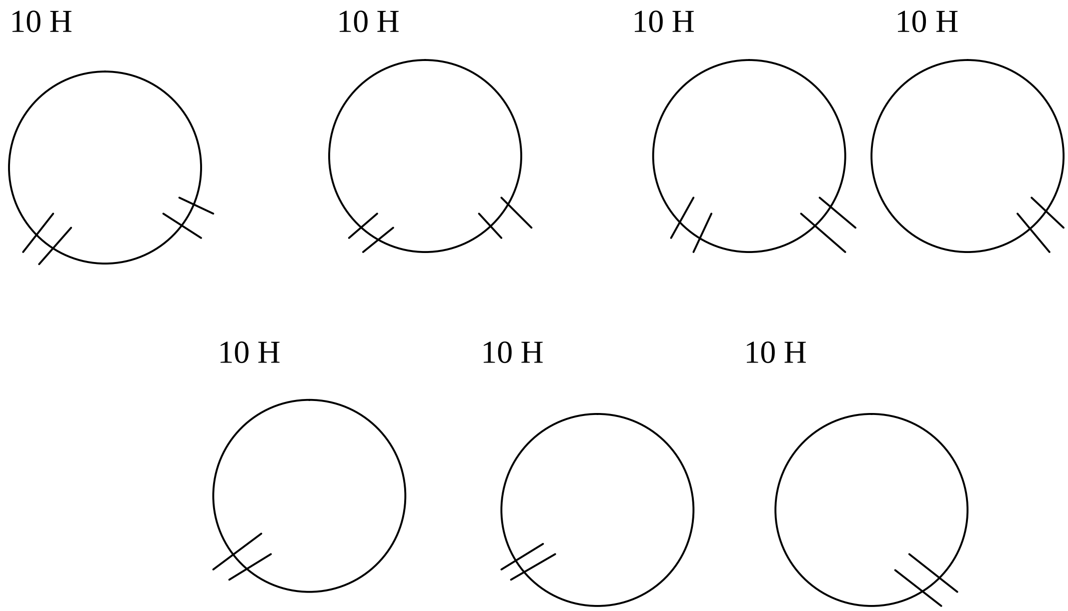
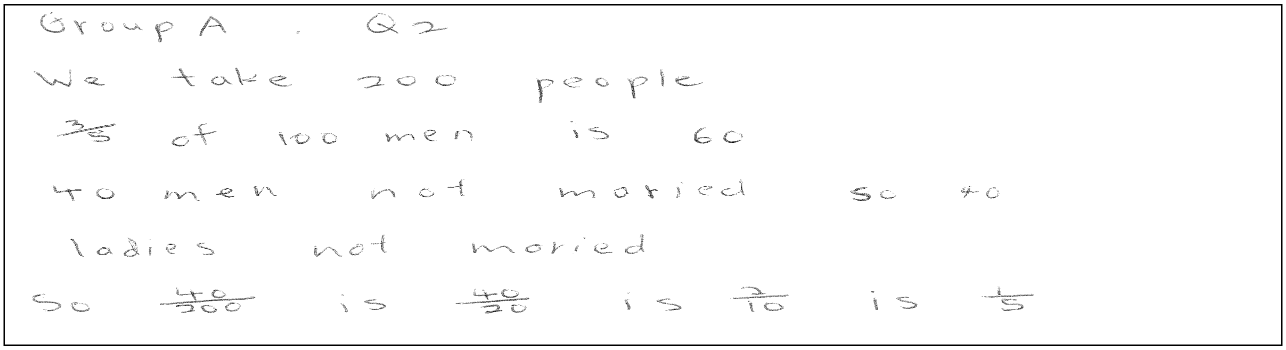


Figure 1: Using diagrams as a problem solving strategy for task 1

This represented a mechanism to assist them when devising a plan. They wrote: *"The first three H have 120 legs where each head contains 4 legs for a goat. The last four H contain 80 legs where each head contains 2 legs for a chicken. Now for* $3H \Rightarrow 30$ *heads and* $4H \Rightarrow 40$ *heads*." This was actually their explanation to the solution of the problem in carrying out the plan. In the end they wrote: "*There are 30 goats and 40 chickens. So 30 + 40 = 70 and 120 + 80 = 200*." This showed to us that they looked back at their findings satisfying the given information. The success in this solution highlights that all Polya's four stages were prevalent during the attainment of the correct answer. From this we can induce a possible didactic cue to implement in our teaching. We could probably ask our learners to actually use these levels as a checklist when solving Mathematics problems. We look at Group A written response to Task 2 in Figure 2. Firstly Group A seemed to have understood the statement of the problem in Task 2. This we can assume as they wrote : "$\frac{3}{5}$ *of* 100 *men is* 60" which implied that they considered an equal split of the couples. Secondly, the plan used by them was an instrumental one based on mental reasoning. They did not, however, look back at the question. This would suggest


Group A . Q2

We take 200 people

3/5 of 100 men is 60

40 men not moried so 40

ladies not moried

So 40/200 is 40/20 is 2/10 is 1/5

Figure 2: An example of a correct written response for Task 2

that the first three stages of PLM are partially responsible for successful problem solving.

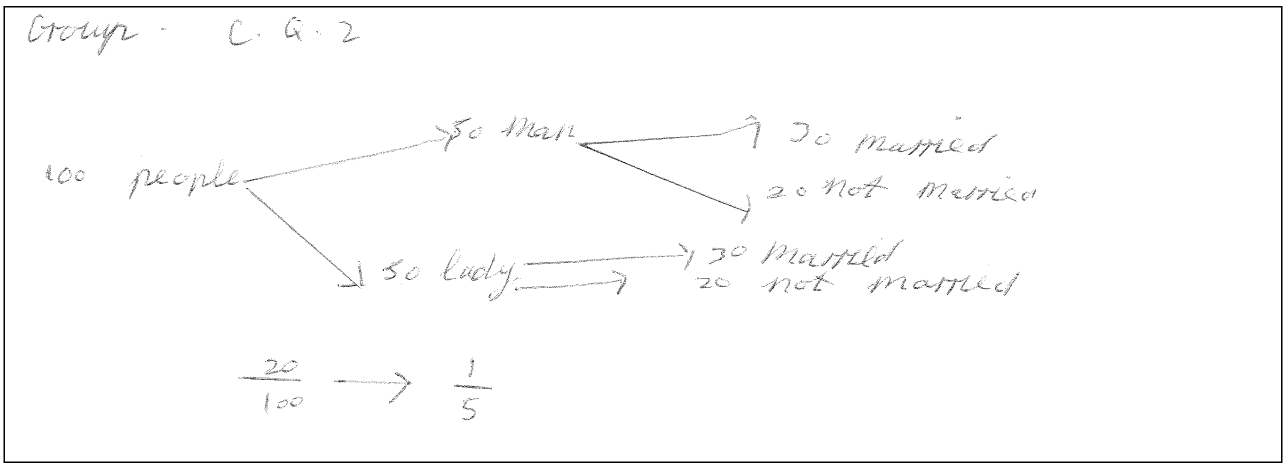


Figure 3: A correct written response by Group C for Task 2

Group C displayed partial responses to Task 1. They understood the problem, they devised a plan which could have worked. They did not satisfy stage four in Task 1. The first three levels were implemented by this group in Task 2 as can be seen in Figure 3. We may assume that this group understood the problem statement for Task 2. The plan they devised involved the use of diagrams and flowcharts. This strategy helped to visualize the solution in a holistic manner. However, they did not look back at the problem. For Task 2, Group D and Group E provided incorrect solutions. They failed to devise a legitimate plan and invariably were on the wrong path. Generally correct solutions displayed the first three stages of PLM. However, the converse of this, namely the first three stages of PLM is no guarantee to completely correct solutions to mathematics problems. The analysis of this data showed that the fourth stage is an absolute necessity for problem solving.

Comments on Polya's stages

Group A had drawn pictures in order to solve problem task one. This concurs with the stages of PLM as Polya (1995) stated that the drawing of pictures was an important aspect in discovery and problem solving. These stages implicitly helped learners cope with problem solving in this context.

Table 5: Individual responses

	Correct responses		**Incorrect responses**		**Completely cannot Respond**		**Partially correct**	
	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
	1			2	4	4		**1**
	2		6	6				
			3	3				
			5					
Total	**2**	**0**	**3**	**3**	**1**	**1**	**0**	**2**

The successful learners looked at the question with understanding, devised a plan, communicated how to carry out plan and lastly, in one case, they looked back. These stages surfaced during the collaborative learning despite the fact that these learners were not taught about the stages of PLM. Although the individual participants displayed some features of the PLM, nobody could solve Task 2 correctly.

The written response of Individual 1 had displayed the four stages of PLM. She understood the question and devised a plan where she had guessed and got the correct answer. She carried out her plan and lastly looked back. In Task 2 the problem has been done but she lacked basic operational (multiplication) skills. We found that the written response for task one by Individual 2 had displayed three stages of PLM. She failed to do Task 2 because it seemed that she did not understand the question. Individual 3 did not understand both tasks. His responses were both incorrect. In his case we could assume that he had no clues as he had no help from his peers. Individual 4 had not responded at all. It seemed that he had no understanding of the demands of the problem Individual 5 had no idea on how to solve task 1 but tried task 2 where she got both fractions correctly and failed to carry out the plan. Individual 6 completely did not understand the task. Both his responses were incorrect.

CONCLUSION AND RECCOMENDATIONS

The situation of very low performance in problem solving has to be overcome by both Mathematics teachers and learners. This can be accomplished in various ways:

a) Development and use of Mathematical skills displayed by learners from other experiences to enhance appreciation and their attitude towards problem solving. This study highlighted this by using tasks which had information from everyday life experiences.

b) Learners should be regarded and accepted as intelligent and creative individuals whose questions are valued. Teachers should therefore afford the learners more choice in discussion. This would give the learners effective decision making powers in problem solving. This can be done by allowing them to work collaboratively. The stages of PLM could assist direct this discussion. Also, they need to negotiate their solutions with their peers in class and be allowed opportunity to take ownership of their learning.

c) Learners should be allowed to use any language to communicate Mathematical ideas, concepts, generalizations and thought processes. This will include the use of language to express Mathematics investigations and interpretation.

d) Teachers must introduce and present problem solving in an interesting and joyful manner. OBE has introduced a new thinking in our education. A suggestion from this research is to employ and instil the stages of Polya's problem solving model. Mathematics teachers should instil in their learners the need of a checklist which can be adopted as a problem solving teaching-learning activity during problem solving in class.

This study was a case study and only considered tasks involving fractions. The results could not be generalised. There is scope for further research in this area involving larger scale action research to determine the effectiveness of PLM in mathematics problem solving. Research involving other mathematics topics should be explored.

REFERENCES

Barkley FE, Cross KP, Major C 2005. *Collaborative learning techniques.* (pp.4). United States of America: Wiley.

Barwell R 2003. Patterns of attention in the interaction of primary school mathematics students with English as an additional language. *Educational Studies in Mathematics*, 53, 33-59.

Brijlall D 2008. Collaborative learning in a multilingual class. *Pythagoras*, 68, pp. 52-61.

Brijlall D 2011. Solving word problems in a multilingual class. *Proceedings of the 19th annual meeting of the Southern African Association for Research in Mathematics, Science and Technology Education*. pp. 50 - 65. North-West University, Mafikeng Campus, South Africa.

Cohen L, Manion L, Morrison K 2007. *Research methods in education. (6th Ed.)*. London: Routledge.

Department of Education 2003. National curriculum statement Grade 10-2. Govt Gazette no. 23406/2002/ Vol 443/ May). Pretoria, South Africa.

Department of Basic Education 2010. *Curriculum and assessment policy statements (CAPS)*. Mathematics grades 10--12. Pretoria: Government Printer.

Hossain MA, Tarmizi AR 2012. Gender-related effects of group learning on mathematics achievement among the rural secondary students. *Eurasian Journal of Educational Research.* 47, 1-22.

Loji K 2012. Towards teaching methods that develop learning and enhance problem solving skills in engineering students. *South African journal of Higher Education*, 26(1), pp. 120-125.

Maharaj A, Brijlall D, Molebale J 2007. Teachers' views on practical work in the teaching of fractions: a case study. *South African Journal of Education*. 27(4), pp. 597 – 612.

Maharajh N, Brijlall D, Govender N 2008. Pre-service mathematics students' notions of the concept definition of continuity in calculus through collaborative instructional design worksheets. *African Journal of Research in SMT Education*, 12, 93 – 108.

Polya G 1957. How to solve it; New York: Doubleday & Co. Inc.

Polya G 1965. Mathematical discovery: *On understanding, learning and teaching problem solving.* (Vol 1). New York: Wiley.

Ramnarain U 2011. Teachers' use of questioning in supporting learners doing science investigations. *South African Journal of Education*. 31, pp. 91 – 101.

Schoenfeld L 1992. Learning to think mathematically: problem solving, metacognition and sense making in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.334-370). New York: Macmillan Publishing Company.

Stears M, Gopal N 2010. Exploring alternate assessment strategies in science classrooms. *South African Journal of Education*. 30, pp. 591 – 604.

Steffe LP 1991. Mathematics curriculum design: A constructivist's perspective. *In international perspectives on transforming early childhood Mathematics Education. Cambridge:* Cambridge University press.

Sun X 2011. "Variation problems" and their roles in the topic of fraction division in Chinese mathematics textbooks examples. *Educational Studies in Mathematics*, 76, 65-85.

Tobias B 2006. Mathematical word problems. *African Journal of Research in SMT Education*, 10 (2). pp. 1-14.

Von Glasersfeld E 1987. Learning as a constructive activity. In C. Janvier (Ed). *Problem of representation in the teaching and learning of Mathematics*. Laurence Erlbaum, Hillside, NJ.

Wellington J 2000. Teaching and learning in secondary science. (pp.65-67). *Contemporary issues and practical approaches*. London and New York.