International Journal of Education Sciences (IJES)
Received on 08. 08. 2014/05. 09. 2014
REF. NO. IJES-511
(Always refer our Reference No. for all correspondence)
The Relationship between Problem Solving and Conceptual Understanding in Grade Six Mathematics
Rankhumise Mmushetji Petrus[1] and Maimane Joseph Ramathibela[2]
Central University of Technology, Free State
E-mail: [1]<mprankhumise@cut.ac.za>, [2]<Jmaimane@cut.ac.za>

The Relationship between Problem Solving and Conceptual Understanding in Grade Six Mathematics

Rankhumise Mmushetji Petrus[1] and Maimane Joseph Ramathibela[2]

Central University of Technology, Free State
E-mail: [1]<mprankhumise@cut.ac.za>, [2]<Jmaimane@cut.ac.za>

KEYWORDS Conceptual understanding, Problem solving, Fractions

ABSTRACT This paper reports the findings of a research study that explored the relationship between problem solving and conceptual understanding in grade six mathematics. A true experimental design consisting of an experimental (n=16) and control group (n=16) was used. The subjects were drawn from a secondary school in Free State Province. A self structured pre and post test was administered to both groups. A t-test was used to analyse the data. The results of the test showed that the use of problem solving had a positive effect on the learners' conceptual understanding of fractions in grade six mathematics.

1. INTRODUCTION

The education system in South Africa is presently undergoing radical changes. The introduction of CAPS suggests that we move away from the traditional way of teaching, especially in Mathematics and Physical Science. Educators are encouraged to adopt constructivist approaches and problem solving. This study investigated the influence of problem solving on conceptual understanding of equivalent, addition and subtraction of fractions to grade six learners. Learners perform badly in mathematics because the mathematics learned in the classroom is isolated from real life experience of learners (Hiebert and Douglas 2007).

Generally in problem solving, there are no fixed rules or methods for the solution of specific problems. Learners can use objects or sketches to test their ideas. When learners are presented with objects and sketches during instruction they showed a lot of enthusiasm. They could solve problems that they could not compute when presented in numerical context. For example most of them could not compute 1-$\frac{3}{4}$, but when this was presented in a real life situation, they could solve it.

According to CAPS 2009, learners must be able to identify and solve problem and make decisions using critical and creative thinking. According to (de Lange 1992:49; Westhood 2004) learning is not a process of passively absorbing information and storing it in easily retrievable fragments as a result of repeated practice and reinforcement. Instead, he continues, students approach each new task with some prior knowledge, assimilate new information, and construct their own meaning and in the process this builds conceptual understanding. Classroom research with grade 8 learners showed an improvement in performance relating to fraction as they appear in the real life situation.

2. Literature Review

Hiebert (2013) noted that successful performance in the conceptual domain is necessary for problem solving in mathematics. According Sai (1994), conceptual domain has several components:

✓ Understanding properties e.g. knowing where and when various numbers apply and how to use number properties to simplify computation.

✓ Perceiving gestalt of mathematics, e.g. recognising which skills and processes can be used to solve particular problems

✓ Communication, e.g. able to paraphrase problem solutions in simple non-technical terms to describe observations about properties and processes

From these components it can be seen that for learners to develop conceptual understanding, they must be exposed to problem solving strategies. When learners solve problems they become aware of the metacognitive processes and as a result develop greater understanding of mathematics (Fortunato et al.1991; Fernadez et al.1994; Sungur and Tekkaya, 2006) As Furtunato et al.1991 stated, “The problems need to be non-routine.” The simple reason being, if algorithms and procedures are very familiar with the learners, they will not have to think about solving problems. Research by (Furtunato et al.1991:38; Luger 2005), found most problem solvers would use natural strategies of drawing a picture or a diagram when solving problem. This was after they have given learners a word problem to solve.As learners solved the problem, with the help of a diagram that they have drawn, they form concepts in their minds. Once learners have formed concepts of any lesson they are studying, they will be at ease to comprehend the lesson. Classroom instruction should be organized around the solving of a large variety of problems e.g 13-5.They state that in the exercise learners are told that they are going to subtract the small number from the big one. These exercises, they continue, will encourage rote computation and not read carefully or think about the problems. The above exercise can be presented as a word problem. For example, Connie has five (5) marbles. How many more marbles does she need to win to have thirteen (13) marbles altogether. Learners think about these problems, not as examples of addition and subtraction, but as unique situations. By modeling this problem. Learners gradually build the concept of whole numbers and natural numbers and hence conceptual understanding as opposed to operational understanding.

According to (Cangelosi 1996:71; Gutstein and Peterson 2005).), real life problems are used to motivate learners to work on the mathematical content of the unit and to help them bridge school mathematics with real world mathematics. His view is supported by (Clarke and McDonough 1989:20; Bentley 2012) and they believe that if we cannot predict the problems which will confront our learners when they leave schools, then our efforts must be directed towards equipping them to meet unfamiliar problems with confidence.

Another research carried out by English (1998) on student's problem posing within formal and informal contexts. He found out that the problem posing program did show significant improvement in learners' ability to generate their own problems, hence conceptual understanding Christou et al.2005). He found the results on a post-test higher than that of the pre-test. Problem solving play an important role in the study of mathematics and mathematics teaching should assume that role (James 1990;Fennema et al.1991; Hiebert & Carpenter 1992;Fernandez et al. 1994). Hiebert and Carpenter (1992) also found that problems such as $4\frac{1}{8} - 1\frac{5}{8}$ were related to situations like sawing $1\frac{5}{8}$ feet off a board. They could solve as many problems within the informal context and when these fractions were introduced formally like above, they could be related to the informal one (Candes and Tao 2005). Martha is to celebrate her thirteen birthday party and she has to invite her friends. The problem she will be facing include how much money does she have on that basis how many people should she invite. To her this is a problem. She might have developed conceptual understanding with the kinds of problems she solved at school unlike operational knowledge only.Polya's heuristic model for problem solving has four stages (Niewoudt 2000; Fernadez 1994; Polya 2014).

Understanding the problem

Make a plan

Execute a plan

Assess the solution

Problem solving offers learners the opportunity to go through these stages when they solve problems. As a result their conceptual understanding is built gradually. Consider, for example, the following exercise, $\frac{1}{8} + \frac{1}{8} = \boxed{}$

Another problem which followed was that, a mother bought a pizza and cut it into eight (8) pieces. She gives Maria a piece and Peter another piece. How much of pizza will be eaten by Maria and Peter? The problems are the same but presented differently .At the end of the day which learners will develop conceptual understanding of the problem? Of course the learner who is faced with a word problem.Firstly she /he should understand the problem, make a plan like drawing a sketch as in Figure 1.

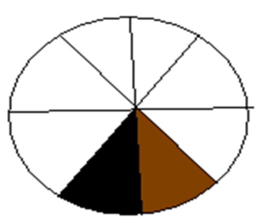

Figure 1

When learners solve problems the use of sketches and models help them to construct meanings to themselves. The use of a sketch help them to form mathematical concepts like fractions, once learners realize that taking apart from a whole will form a fraction, they will understand the application of fractions in a real life context. This process is known as mathematising.

3.1 Aim of the study

- ✓ To determine whether there is a relationship between problem solving, as an approach to teaching and learning of mathematics, and conceptual understanding.
- ✓ To find the nature of the relationship between problem solving and conceptual understanding

3.2 Research questions

Again this background, this research is aimed to seek answers to the following questions:

- ✓ Is there a relationship between problem solving, as an approach to teaching and learning of mathematics and conceptual understanding?
- ✓ What is the nature (if there is any) of the relationship between problem solving and conceptual understanding?

4 Methods

4.1 Research Design

The experimental group was instructed in problem solving whereas the control group will not. The research problem is more concerned with the cause and effect kind of a relationship. Therefore, problem solving was independent variable whereas conceptual understanding relied heavily on the method employed and as a result was taken as the dependent variable. A true experimental design consisting of an experimental and control group was used. Pre-testing and post-testing were done and for the sake of validity, the same test was given both as a pre-test and post-test.

4.2 Subjects and method of sampling

The subjects were drawn from a school in the Free State province. The school has only two classrooms of grade six. With the toss of the coin grade 8a (n=16) became my control group and grade 8b (n=16) was my experimental group. The approximate ages of learners in those two classes were eleven and twelve years. The method sampling used is random selection. Each class consisted of learners with a mixed range of abilities. The learners had experienced a range of been exposed to the problem-posing tasks of the types included in the investigation. The test questions that were given to learners were aligned with grade six in the intervention.

4.3 Measuring instrument

A self–structured mathematics test used as a pre and post-test. Test validity was maintained by carefully selecting test items that would measure learners understanding of equivalence, subtraction and addition of fractions. The pre-test results were used for diagnostic purposes, and the post-test results were used mainly to provide a measure of change in performance of addition and subtraction of fractions after instruction in problem solving.

4.4 Procedure

A pre-test was administered in both groups i.e. control group and experimental group. To see how effective problem-solving was on both groups wrote the same test. The test included equivalent fractions, additions and subtractions of fractions. The experimental group received instruction in problem-solving while the control group was taught the same lesson, but in a traditional way. The teaching programs consisted of three teaching episodes that lasted for about two weeks. The following is a list of topics, with the number of lessons in brackets.

- ✓ Equivalent fractions (two lessons)
- ✓ Addition of fractions (three lessons)
- ✓ Subtraction of fractions (three lessons)

The class divided into six groups with each group consisting of three learners. Each group was given two pictures of a cake as shown in Figure 2.

a **b**

Figure 2

The learners were presented with this question. A mother bought two cakes for her sons. She cut one of them in two pieces and the other into four pieces. She gave John one piece out of two and Peter two out of four pieces. Between Peter and John who will have a bigger share? (Assuming that the cakes are equal in size). Learners worked in groups and the researcher was moving around listening to their discussions. They could see that in fact, $\frac{1}{2}$ is the same as $\frac{2}{4}$ and all of them agreed that they are equal.

Apart from $\frac{2}{4}$ equalling to $\frac{1}{2}$,there are also fractions, which are equal to $\frac{1}{2}$. The researchers designed fraction table from (Johnson et al.1999).

This was cut into pieces and each group was given a set of piece to work with. He asked them to combine different pieces that will be equal to $\frac{1}{2}$. The researcher was moving around the groups listening to their inputs and some groups see that $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}$. However some made a common mistake by saying for example $\frac{1}{2} = \frac{1}{6}$. They only counted a sixth instead of three. After some intervention they could see that $\frac{3}{6}=\frac{1}{2}$ and not $\frac{1}{6}$ and they could do the same with $\frac{1}{3}$ and so on.Now about "bigger" fraction, the researcher asked them to solve this problem e.g. $\frac{1}{2} = \frac{25}{}$

The second topic was about addition of fractions. The researcher did not want to present them (addition and subtraction) simultaneously because they might be confusing to learners. He used a drawing of a cake being cut into four pieces as shown in figure. Once again this model was given to each group and after they had successfully solved the problem, other models with different pieces were given to them to solve.

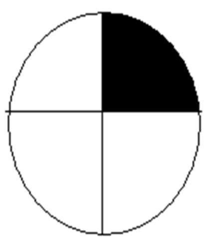

Figure 3

The pieces were coloured differently and it has to be shared by four children. If Betty gets the blue piece and Sam receives the red one, how much of the cake is eaten? So, it will be $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$.It is important to start with the problems where the denominators are the same. During the pre-test, when learners were given similar problem. Some even added the denominators for example, $\frac{2}{5}+\frac{1}{5}=\frac{3}{10}$. When presented with a sketch, or when they drew their own, they got it right. Of course, one or two group repeated the mistake and the researcher referred them back to equivalent fractions. He asked them to simplify $\frac{2}{8}$ and after a long workout, they got the answer as $\frac{1}{4}$

The next lesson was on fractions with different denominator and learners could apply equivalent fractions to add and subtract fractions. These problems were not presented as exercises to learners but as everyday life problems. At the end of the two weeks of the presentation the same pre-test was given to both experimental and control groups as a post-test to determine the effect of problem solving on conceptual understanding.

As indicated above, the examples that were used in class were real life ones. As a result it generated interest among learners that they forgot that they were learning mathematics. For example, cutting a pizza into pieces and share it amongst members of the family it was so interesting because some of them ate pizza the previous day.

5. Statistical techniques and analysis of data

The t-test used to analyze the results of both the pre and posttests.

5.1 Pre-Test and Post-Test results

The post-test scores for both experimental and control groups were recorded. The implementation of the teaching program occurred as planned but it appeared that I need at least another week to be sure that every learner has been given attention. The use of sketches, diagrams and models helped learners to interpret a problem correctly. Pre- and post-test result indicated some changes in learner's performance in equivalent, addition and subtraction or fractions. Since the researchers were investigating the effect of the problem solving on conceptual understanding, a one tailed test was adopted.

The value of t during the pre-test was found to be $t=1.07$. The degrees of freedom (d f) =16=16-2 = 30. The criteria for $p<0.05= 1.6973$ and $p<0.01= 2.4573$ (one tailed test), therefore the calculated value of $t=1.07$. This value is less than the value for both criteria of p. in this instances the null hypothesis will be accepted. In other words this means that there was no difference between the mean values of the groups before post testing was done.

The same learners took part in the post-test as well. It was found after calculations that $t=2.71$. Using the table for criteria value of t, where the degree of freedom is 30, the criterion for $p<0.05 = 1.6973$. Again the criterion for $p<0.01 = 2.4573$ and since $t= 2.71$ is larger than both the criteria for $p<0.05 = 1.6973$ and $p<0.01 = 2.4573$, a conclusion can be made that there is statistically significant difference in the understanding of fractions between the experimental and control groups. The null hypothesis is rejected because there is difference between the mean of the two groups. This now implies that the use of problem solving had a positive effect on the leaner's conceptual understanding of fraction in particular and mathematics general.

6. Summary and conclusion

This study has shown how problem solving can be used in teaching fractions in grade six. Using this method in a real classroom situation has demonstrated its effectiveness in building learners conceptual understanding in mathematics. Learners understood fractions better when they drew sketches and diagram. The control group, apart from repeated reiterations about not adding the denominator, still continued to add them. For example $\frac{1}{4}+\frac{1}{4}=\frac{2}{8}$. It has been noticed that experimental group would draw a figure, divide it into equal parts and easily got the answer. In the beginning learners did not understand that one over two ($\frac{1}{2}$) is the same as half ($\frac{1}{2}$). To them these two concepts were different from one another until this was presented as a real life problem. For example cutting a half loaf into two equal parts. I would have been happier if I had more time at my disposal. The two weeks that I used problem solving produced some changes in learners understanding. The changes might have been brought about the fact that they wrote the test (post) whilst they could still remember what has been taught. Problem solving has increased learners conceptual understanding in fractions. Further research is required to investigate the influence of problem solving in geometry as it is about shapes and figures which learners interact with in their everyday lives.

Bibliography

Babbie E 1996. The practice of social Research. 7 th ed. California : Wadworth

Babbie E 1999. The basics of social Research. 8th ed. Belmont : Wadworth

Baily KD 1999. Methods of social Research. 3rd ed. London: Macmillan.

Bentley T 2012. *Learning beyond the classroom, Education for a changing world*. Routledge.

Candes E J,Tao T 2005. Decoding by linear programming. *Information Theory, IEEE Transactions on*, *51*(12): 4203-4215.

Christou C, Mousoulides N, Pittalis M, Pitta-Pantazi D, Sriraman B 2005. An empirical taxonomy of problem posing processes. *ZDM*, *37*(3), 149-158.

Clarke D,McDough A 1989. The problems of the Problem Solving classroom. Australian Mathematics teacher, 45(3):20-24

Dole S 2000. Promoting Percent as a proportion in eight-grade Mathematics. School Science and Mathematics, 100(7):(10p.),Mar. [In EBSCOHost : Academic Search Premier, Full display : http:www-sa.ebsco.com][Date of acess : 04 April 2003].

English LD1998. Children’s problems posing within formal and Informal context. Journal for Research in Mathematics Education, 29(1):(24 p.), Jan. [In EBSCOHost : Academic Search Premier, Full display : http:www-sa.ebsco.com] [Date of access : 25 March 2003]

Fernandez ML, Hadaway N, Wison JW 1994. Problem solving: managing it all. The mathematics teacher, 87(3): 195-199.

Fortunado I, Hecht, D, Tittle CK, Alvarez L 1991. Metacognition and problem Solving. Arithmetic teacher, 38-41.

Garbers JG 1996. Effective Research in the Human Science. Pretoria: Pretoria Van Schaik.

Gooard W, Melville S 1996. Research methodology: An introduction for science and Engineering students. Cape Town: Juta.

Gutstein E, Peterson B 2005. *Rethinking mathematics: Teaching social justice by the numbers*. Rethinking Schools.Handbook of research on mathematics teaching and learning. New York : Macmillan

Hiebert J.2013. *Conceptual and procedural knowledge: The case of mathematics*. Routledge.

Hiebert J, Douglas A 2007. "The effects of classroom mathematics teaching on students’ learning." *Second handbook of research on mathematics teaching and learning* 371-404.

James N 1990. Investigate approaches to the learning and teaching of mathematics. Pythagoras, 24:11-16

Johnson Y, Baxter M, Coombe J, Callant J 1999. Maths for all Manzini: Macmillan BOleswa.

Leedy PD, Ormrod JE 2000. Practical research. Planning and design. 7th ed. Upper Saddle River, N.J: Prentice Hall.

Luger GF 2005. *Artificial intelligence: structures and strategies for complex problem solving*. Pearson education.

Mulder JC1993. Statistical Techniques in Education. Pretoria: Huam Tertiary.

Nieuwoudt H D 2000. Approaches to the teaching and learning of mathematics. 2nd ed. Potchefstroom.

Polya G 2014 . *How to solve it: A new aspect of mathematical method.* Princeton university press.

Rossouw D 2002. Intellectual Tools: Skills for the Human Science. W.Cape:

Sai KP 1994. Doing and talking in Primary mathematics. *Pythagoras*, 34:15-19.

Sungur S, Tekkaya C 2006. Effects of problem-based learning and traditional instruction on self-regulated learning. *The journal of educational research*, *99*(5):307-320.

Westwood P 2004. "Learning and learning difficulties: A handbook for teachers." *Learning and Learning Difficulties: A Handbook for Teachers*.