The Application of Algebra in the Determination of Equilibrium Wage in Labour Market: Nigeria

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KEYWORDS

ABSTRACT
Algebraic models are more endowed to economists than the traditional theories and diagrams from which they are derived, being more precise and clearer. Using algebra, there exist equilibrium wage and unemployment in labour market, through the age long A.W. Philips curve. Then, the question of sensitivity is the extent of unemployment that must be tolerated to ensure price stability. The objective of the paper is to show that algebra is effective in determining equilibrium wage and unemployment, and it recommends among others, a policy of price control, so that the negative effects unemployment and inflation have on wage can be neutralized in the different periods.

INTRODUCTION

In 1958, Alban William Philips, answered the question that has caught the attention of economists on whether unemployment reduction can result in price instability, by giving empirical evidence from the British economy. He developed a trade off between unemployment reduction and less inflation, and confirmed this fear, on the antagonism between these two important macroeconomics goals (Manuel 2006; Mankow 2007). One reason why this could be right is that the increased wage from increased employment increases aggregate demand in the product market, forcing up prices. Real Time Economics, in the Wall Street Journal of January 27, 2009, in an article titled: “Philips Curve Makes Ugly Comeback”, warned that it is scary to watch it play out. Richard Lipsey, a Canadian Economist, found in Lipsey (1965), Carlaw and Lepsey (2006), elucidated this relationship since Philips lacks details of empirical evidence from the British economy. Hence, the Lipsey-Philips curve, the theoretical analysis, from where this algebraic determination of equilibrium in labour market, is derived (also see in Bruce 1987). This is even more absurd in Nigeria, as a developing country, where partial coverage and other imperfections exist in the labour market.

The research makes use of algebra in the main issue of equilibrium determination between wage and unemployment in the labour market. That is, at what point in the labour market is equilibrium determined between acceptable unemployment and wage, without leading to unacceptable inflation? It also follows other writers to apply quantitative techniques on issues militating against the labour market in Nigeria. It does not present any data and does not actually estimate any variable, since it is an expensive and a different research to apply the equilibrium models in this research to an empirical study of the country. This means that it remains an estimation tool for actual empirical research.

A THEORITICAL FOUNDATION OF EQUILIBRIUM AND ALGEBRA

On the issue of equilibrium and algebra, Iyoha et al. (2002), in applying matrix Algebra to the problem of optimization in managerial economics, posits that the device of algebra permeates the solution technique to economic problems especially when the decision variables are many. These authors believe that the technique of linear algebra offers a clear and manageable notation for the formulation of economic model involving many decision variables. They see equilibrium as generally referring to the attainment of a certain price from which there is no tendency to move because the economic unit concerned prefer that price, or it is a state of balance between conflicting forces operating in the market (Iyoha et al. 2002). This analogy is not different
in the labour market, as the prices of goods in the product market is comparable to the wage of labour in the labour market, since they both assure the supplier the satisfaction to part with goods and labour respectively, given the prevailing markets conditions. Benrdett (2006) also recognized this existence of equilibrium in the labour market when the author was concerned with the human capital accumulation and labour market equilibrium. Moreover, the two markets fulfill the Walras law of aggregate demand function in that they satisfied a non-positive aggregate excess demand (See Starr 1997; Balasko 2009). Starr further contends that in proving the existence of equilibrium, we must develop a mathematical structure to establish the premise for the existence of such equilibrium. Some major contributors in this direction are Arrow (1974), Arrow Debrew (1954) and recently, Balasko (2009) in his “General Equilibrium without Utility Function: How Far to Go?” Hence, a departure from an established works of authors and algebraic manipulation to determine equilibrium is not quiet novel to this research.

According to Bruce (1987), the popular Philip curve was expatiated by Richard Lipsey, a Canadian economist because the relationship between wage and unemployment as propounded by Philips lacks detail of empirical evidence of the British economy. This was reflected in Carlaw and Lepsey (2006), which brought to focus the Lipsey-Philips curve from which this algebraic determination of equilibrium is derived. Lipsey had developed a relationship if there is a decrease in wage, which is most likely in Nigeria because of many market imperfections. The Lipsey relationship is as follows:

\[ W = b [LD - LS] \]

He takes LD and LS to be labour demand and labour supply respectively, while W is real wage and b, a multiplicative constant. This determines the relationship between wage and unemployment, and then extends to equilibrium in the economy, using the rate of change in money wage of the consumer price index (CPI).

A BRIEF REVIEW OF EQUILIBRIUM PROOF AND DETERMINATION

We begin with two cases of the use of algebra to determine/prove equilibrium existence in either markets, since there are very close similarities with both the product and labour markets-the process of equilibrium determination is through the same interactions between demand and supply, and excess aggregate demand in both markets is non-positive in the case of general equilibrium– The Walras’ law (see also Balasko 2009)).

a. Quirk and Sapoznik (1968) had done a good work on the existence of comparative market equilibrium. Diasakos (2008) also recently gave several proofs of the theorem even with cases of uncertainty in the economy in his comparative static of general equilibrium asset pricing. This is as follows:

(i) \( x^e \in X^e; y^e \in Y^e; m = 1, ..., m, e = 1, ..., L \)

Consumers maximize utility subject to the constraint of their income.

\[ \sum_{m=1}^{M} x^e_r \leq \pi^m + \sum_{e=1}^{L} \pi^e \]

(ii) \( y^e \geq \pi^e \)

Producers maximize profits subject to their production function

\[ \text{i.e. } \sum_{m=1}^{M} x^e_r - \sum_{l=1}^{L} y^e_l = r \]

They then posited that Walras’ Law is satisfied as aggregate excess demand is non-positive (\( \leq 0 \)).

b. Ekanem (1998) and Monique (2009), proof of the existence of general equilibrium began with the application of the traditional 1956 and 1955 Gale-Nikaido approach in a pure exchange model and then proceeded to the introduction of productive activities. The authors then summarized respectively their works in the equations, which characterizes the state of the equilibrium as follows:

(i) \( x^e \in X^e, y^e \in Y^e; m = 1, ..., M; l = 1, ..., L \)

Consumers choose within the attainable state of economy in such a manner as to maximize their utilities subject to the constraints of their income.

(ii) \( \bar{y} \in y^e/p_y \geq P_y^e \)

This, he said producer maximize profit subject to the aggregate products set.
(iii) \( \sum_{m=1}^{\infty} x_m - \sum_{m=1}^{\infty} y_m = r \) \( \text{i.e.} \ \sum_{m=1}^{\infty} x_m - \sum_{m=1}^{\infty} y_m - r \leq 0 \)

This equation (iii) is consistent simultaneously with equations (i) and (ii), and equilibrium is satisfied because it fulfils Warals Law.

Table of Notations

- \( f \) functional notation
- \( \in \) is an element
- \( \forall \) for all
- \( \geq \) greater than equal to
- \( \leq \) less than equal to

ALGEBRAIC DETERMINATION OF EQUILIBRIUM WAGE LABOUR MARKET

Having satisfied the curiosity of our authority in using algebra to prove equilibrium, we proceed to determine a single state of such equilibrium in the labour market.

Recall our Lipsey relationship as:

\[
W = b \left[ L_D - L_S \right] \quad \text{.................. (1)}
\]

Lipsey (1965) has the following relationship as Lipsey-Phillips curve:

Real wage/Inflation

\[
W_1 \quad W_2 \quad W_3
\]

Labour

Unemployment will rise at excess demand \( L_{D1} - L_{S2} \), say at a disequilibrium \( W_1 \). But firms raise wage from \( W_1 \) as an option to attract workers. Thus, Increased \( L_{D1} \) increases \( W \) where: \( L_{D1} \) = excess demand for labour and \( W \) = Wage.

The rate of increase in real wage at \( W_1 \) is greater than \( W_2 \) since \( L_{D1} - L_{S1} > L_{D2} - L_{S2} \)

Where: \( L_{D1} \) = Labour demand before the increase in wage

\( L_{S1} \) = Labour supply before the increase in wage

\( L_{D2} \) = Labour demand after the increase in wage

\( L_{S2} \) = Labour supply after the increase in wage

Where supply equals demand at \( W \), there would be equilibrium. Unemployment will decrease as excess demand increases. But if supply is greater than demand at say \( W_1 \) (a wage above the equilibrium, \( W_e \)), unemployment will result, especially in periods of fixed wage, when the increase supply cannot force down wage for firms to be able to employ more workers.

If there is a decrease in wage, say from \( W_3 \) to \( W_2 \) or \( W_1 \) to \( W_0 \) in our preliminary diagram

\[
\bar{\omega} = b \left[ L_D - L_S \right] \quad \text{.................. (1)}
\]

Where: \( \bar{\omega} = \text{real wage} \), \( b = \text{multiplicative constant} \), \( L_D = \text{labour demand} \) and \( L_S = \text{labour supply} \). This determines the relationship between wage and unemployment, and extends to their equilibrium, using the rate of change in money wage of the consumer price index (CPI).

Let frictional unemployment resulting from supply > demand in preliminary diagram be \( \mu \)

Then, \( \mu = \bar{\mu} - a \left[ L_D - L_S \right] \quad \text{.................. (2)} \)

Interpretation:

(i) At excess demand = 0, unemployment = \( \mu \)
(ii) At excess demand > 0, unemployment < \( \mu \)
(iii) At excess demand < 0, unemployment > \( \mu \)

We can combine equations (1) and (2) above.

\[
\bar{\omega} = b \left[ L_D - L_S \right] \quad \text{.................. (3)}
\]

Where: \( \bar{\omega} = \text{real wage} \), \( b = \text{multiplicative constant} \), \( L_D = \text{labour demand} \) and \( L_S = \text{labour supply} \).

We can combine equations (1) and (2) above.

\[
\bar{\omega} = b/a \mu - b/a \mu \quad \text{.................. (3)}
\]

Where: \( a, b, \) and \( \mu \) are constants

Hence \( \bar{\omega} = \alpha - \dot{\mu} \), where: \( b/a \mu = \alpha \) (intersect), \( b/a = \dot{\alpha} \) (slope).

(Ojameruaye et al. 2001; Iyoha et al. 2002). Thus, real wage is negatively related to unemployment in the simple model.

With the interest being in relationship between unemployment and inflation, we can exploit the rate of change in money wage of the CPI to equation (4), that is:

\[
(\alpha - \dot{\mu}) + \dot{P} = \dot{w} \quad \text{where:} \quad \dot{P} = \text{rate of change in money wage of the CPI.}
\]

\[
\dot{w} = \alpha - \dot{\mu} + \dot{P} \quad \text{.................. (5)}
\]

where: \( \dot{w} = \text{rate of change in money wage.} \)

Since unemployment contract takes length of time, we anticipate the relevant price changes:

\[
\dot{w} = \alpha - \dot{\mu} + \Delta \dot{P} \quad \text{.................. (6).}
\]

a multiple algebraic model. (Ilopuotafe 1999; Ojameruaye et al. 2001). Where: \( \Delta \dot{P} = \text{expected rate of inflation in the next period.} \)
If we create one for each period, at 0,
\[ \hat{w} = \alpha - \hat{\mu} + \hat{\omega}(0) \]
That is, equation (4), the simple model
At \( t = 1 \),
\[ \hat{w} = - \hat{\mu} + \hat{\omega}(1) \]  …………… (7)
At \( t = 2 \),
\[ \hat{w} = \alpha - \hat{\mu} + \hat{\omega}(2) \]  …………… (8)
In time series research, equation (6) can be estimated with the Ordinary Least Square (OLS) econometrics technique to obtain numerical values for \( \alpha, \hat{\mu} \) and \( \hat{\omega} \). Equation (8), from the base equation (6) is highly satisfactory to labour economists in determining equilibrium between wage and unemployment in the labour market since the interest is only on wage inflation. The fulfillment of Warals law is not required since the interest is not on aggregate excess demand, but a single state of equilibrium.

**SPECIAL CASE OF NIGERIA AS A DEVELOPING COUNTRY**

Since according to Ayanwu (1993), Adelakun (2007), and Emejor (2010), there is often rise in price level in the LDCs and Nigeria, \( e \) in equation (6) above is more likely to be a rise in price for the different periods. This implies a too often distortion of this equilibrium. There are other contemporary imperfections in the case of the Nigeria, which are factors responsible for the too often distortion in this equilibrium, needing policy interventions:

**Increasing Demand for Leisure:** This is a cumulative causation of the “working poor” in the developing countries, especially those in government employment whose “take home pay cannot take home.” Parkin (1997) and Shores and Scott (2007) believe that these workers are discouraged. This makes them prefer leisure to work since they could not account for income earned from past work. Therefore, the traditional Robbins Income-Leisure model with the application of indiffenent curves set in since the rational individual tries to attain the highest indiffenent curve possible, which represents the highest utility. The increase demand for leisure further increases poverty level, thus low capacity for production and low wage below the existing equilibrium. A constraint condition for the attainment of utility is implied as (also Shore and Acott 2007):

\[ U = f(I, t, \hat{\omega}) | S.t h(T, \hat{\omega}) = Z \]  …………(1)

Using the Langragian function,
\[ U = f(I, t, \hat{\omega}) + \hat{\omega}(Z - h(T, \hat{\omega})) \]  …………(2)
Where: \( U \) = utility, \( I \) = income, \( t \) = hours of leisure
\( T \) = total hours of leisure enjoyable and
\( \hat{\omega} \) = hourly wage.

We now take their partial derivatives to arrive at equilibrium
\[ \frac{\partial U}{\partial I} = f_I - \hat{\omega} = 0 \]  …………(3)
\[ \frac{\partial U}{\partial \hat{\omega}} = f_{\hat{\omega}} - \hat{\omega} \]  …………(4)
\[ \frac{\partial U}{\partial t} = Z - h(T, \hat{\omega}) = 0 \]  …………(6)
Equating equations (4) and (5), \( U_t = U_{\hat{\omega}} \)
\[ \frac{\partial U}{\partial I} = f_I - \hat{\omega} \]
\[ \frac{\partial U}{\partial \hat{\omega}} = f_{\hat{\omega}} - \hat{\omega} \]
(See Guignard 2007)

However, the optimal combination of income and leisure, found in where the marginal utility (MU) of an extra amount of income \( I \) earned in that hour is distorted by the increasing demand for leisure, as a result of low wage.

**The Strong Backwash Effects:** This overshadows the “spread effect”, and leads to further disadvantage of the already less developed localities, especially with respect to labour mobility in terms of occupation and age, to the more developed regions, putting pressure on labour market in those regions. This means a shortage in agricultural sector, leading to inadequate production capacity and ultimately low wage (Jhingal 2000; Watkin et al. 2005).

**Trade Unionism and Implicit Contracts:** These perpetuate wage above equilibrium and result to retrenchment, increase labour supply and force wage down below equilibrium if firms must break even. (Yusuf 2000; Joha-Torsten 2010).

**The International Demonstration Effects:** As Nurkse (1969) sighted in Kottis (2007), had put it, exposures that lead to “higher consumption without a corresponding increase in production” for the developing countries. This reduces the chances of employable factors of production to be fully utilized. The consequence is increase in labour supply, pushing wage below the equilibrium.

**The Insider-Outsider Crisis:** The insiders, defined as those with jobs do everything possible to protect their jobs since they are not sure of another. This disfavours the outsiders, defined as those without jobs. The insiders capitalize on existing employment implicit contracts to hold firms to ransom; high compensation for layoff, job specialization, cost of new employment, settlement allowances and insurance for new
worker and then cost of training, all fixed wage above equilibrium for the working few. Then the only option left for the firm is retrenchment in “bad periods”. This again increases labour supply in the entire industry (Lindbeck et al. 1988; Solow 1995; Kling 2010).

The Wide Partial Coverage in the Labour Market: Both the high and the low income earners go to the same product market for goods and services. Thus, the inadequacies in the labour market are not compensated for in the product market. Workers’ interest is on real wage (Pricing 1997; Carmelo 2008), the low income earners are further disadvantaged in terms of inequality.

The General Rise Price Level: This reduces real wage and workers will not be subjected to money illusion. So they demand for increase in wage above equilibrium leading to economic crises and workers retrenchment. This increases labour supply and ultimately forces down wage (Anyaele 1987; Adelaku 2007).

General Poverty: This does not allow many workers to increase their capacity for higher wage. Therefore, the bulk of labour in Nigeria is unskilled and of low quality, meaning low wage (Ogwu 2007; United Nations 2009; AUPETURE 2010).

Underemployment: This is when a skilled worker is not employed according to his full potential because of excess labour supply (a labour surplus economy), making his wage fall below an ideal equilibrium (Anyaele 1987; Isidore 2008).

Imperfect and Asymmetric Information: This favours the firm who has advantage of the “going around” over the workers who has no insider around him (Lindback et al. 1998).

Complex Price of Wage Rate: This is when wage serves a number of functions, especially in an economy where there is high dependency ratio (Yusuf 2000; Meltezer 2009).

Others are segmentation and class struggle, age, gender discrimination, capital freight, labour immobility, brain drain and more emphasis on strong sustainability than weak sustainability, which all militate against a stable labour market, with most of these factors reducing wage below an existing equilibrium. (See Anyaele 1987; Yusuf 2000; Neumayer 2009).

CONCLUSION

From literature, it was discovered that of General Equilibrium exists. From the Lipsey-Philips trade off between employment and inflation, an algebraic model for determination of equilibrium in the labour market is possible, and was reached in this work, even though many factors lead to the too often distortion of this equilibrium in the Nigeria. It is, therefore, recommended that government of Nigeria come out with high policy interventions to regulate and stabilize the labour market, so that such equilibrium would be stable, and to resist rise in prices in the product market in the next periods.

RECOMMENDATIONS

These come through regulatory policies intervention to stabilize the labour market and wages:

Price Control: To resist inflation and neutralize the negative effect of unemployment, anticipated inflation on real wage since the expected rate of change in price in the next period (e) is mostly likely to be a rise in price for Nigeria. This should be meant to be bias in favour of the outsiders, who are without jobs, or the low wage earners.

Attractive Civil Servant: There should be high and unannounced emoluments, with less politicized benefits for workers, in order not to lead to wage spiral inflation, causing money illusion and reduction in real wage. The demand for leisure will reduce and equilibrium between income and leisure will be maintained.

Egalitarian Policies: Government of Nigeria should strengthen the spread effects and weaken the backwash effects so that the supply of labour and demand for labour would be evenly distributed to the regions. Firms can be given incentive and disincentive policies to go to the less developed regions, rather than concentrating in the more developed regions.

Capacity Building: There should be maximum investment in capacity building and utilization so that all the incentives for fixed wage by either the few workers (insiders) or firms would be removed. Labour supply will then equal labour demand. This alleviates poverty, with workers raising capacity for higher wage. We thus avoid capital freight and brain drain.

Improvement in Communication: This is to remove all imperfect and asymmetric information between the workers and the firm in the labour market. Hence, both the insiders and the
outsiders, including the firm would have perfect information about the “going around” in the labour market.

**Birth Control:** This is to reduce dependency ratio and minimize the complex price of wage rate.

**Abolition of Obnoxious Traditions:** This is to remove all discriminations against gender in employment and remove all customs leading to excessive age discrimination, widen income gap and thereby reducing general poverty.

**REFERENCES**


