Mathematics Teaching Via the Lesson Study Model

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ABSTRACT

The paper presented empirical work related to effective teaching of mathematics in order to determine major issues of importance for future research and to understand the issues in relation to theory and application of Lesson Study Model (LSM) in South Africa context. The study applied two-phased sequential mixed methods. In the first phase, analysis of MANOVA and repeated-measures ANOVA were done to investigate whether there was a significant difference or not between groups in respect of experimental and control groups. The two main findings included (a) LSM was a better predictor of improving mathematics teaching and (b) distinct views on LSM could be identified by the mathematics teachers in the process of using LSM. One of the implications from the study was that LSM could be accepted as a turning point in developing the metacognitive skills, emphasising the reflective teaching and learning and providing internal consistency of instructional planning.

INTRODUCTION

Both past and present research (Ross and Bruce 2005; Takahashi et al. 2006; Takahashi 2007) in mathematics have lamented over effective knowledge of mathematics both in teaching and learning. The authors suggest that mathematics education research have be taken for granted that effective mathematics teaching requires sound content knowledge as well as knowledge of the process of pedagogy; suggesting that teaching and learning of mathematics are confined to both transmission (teaching) and acquisition (learning) of mathematical concepts. The view that lack of understanding of transmission and acquisition of mathematical concepts was further noted by writers (Kim and Baylor 2007; Takahashi 2007), who implied that the argument about the interphase between transmission and acquisition of mathematical concepts is a major problem in that most poor national results in mathematics emanates from lack of attention from above. This concern is particular in South African mathematics education (Takahashi 2007).

The concern points to one main issue, thus, the understanding that effective pedagogy requires both extensive content knowledge as well as pedagogical knowledge from teachers. Consistent with the above, review of South African literature suggests that there is still a disjuncture between extensive content knowledge as well as pedagogical knowledge (Takahashi 2007). Other international literature (Kajander and Lovric 2005) has raised similar concerns, suggesting that the concern appears to be a global concern.

It was in this vein that the Japanese developed a remedial model to better understand transmission and acquisition of mathematical concepts, thus improving mathematics teaching and learning. This model was called the lesson study model. As suggested by the lesson study model literature (Chokshi and Fernandez 2004) the model has been a success story both in Japan and United States of America. The authors suggested that LSM is a teacher directed professional development model developed in Japan. It is grounded in research and not like traditional lesson planning. Hence, the lesson process begins by teachers and other stakeholders looking critically at their mathematics curriculum in a manner that alludes to continues professional development in mathematics.

Thus teachers and stakeholders need to look at their curriculum in their schools in relation to what they know about their student’s learning. This suggests a two-dimensional view involving: (a) looking at both what learners are having troubles learning and at (b) how these concepts are being taught; implying that prospective teachers, as well as teachers, collaboratively engage in meaningful dialogues about answers to such views. It is in this connection that this study adopts and adapts LSM to apply in South African context.

Development of Lesson Study Model

It is imperative to note that in applying this model of lesson study in South African context,
the study retains essential features of the Japanese model, making necessary changes to adapt to the contexts and purposes of South Africa higher education. Thus, whether in Japan or the South Africa, lesson study involves a small team of instructors (researcher and the candidate teachers), working together to design, teach, study, and refine a single class lesson. This work culminates in at least four tangible products; the first including a detailed usable lesson plan, second is an in-depth study of the lesson that investigates teaching and learning interactions, third is explaining how students respond to instruction, and fourth is how instruction might be further modified based on the evidence collected. In this direction, the process adopted and adapted LSM by (a) formulating learning goals or objectives (b) designing the research lesson (c) designing the study (d) teaching and observing the research lesson (e) analysing the evidence (f) repeating the process (g) documenting the lesson study.

Noting that aspects of lesson study resemble other teaching improvement strategies such as backward design and classroom assessment, a closer look at how the lesson study process plays out in higher education, however, reveals important differences with other teaching improvement activities in the South Africa. The difference suggests that to implement LSM, the researcher(s) ought to defining the problem, plan the lesson, teaching and evaluating the lesson and reflect on its effects and finally revise the lesson. This process implies that lesson study is one component of a system designed for continual professional development.

In Japan, lesson study is either done by teachers across a district, or by teachers within a school. But in this study, both the researcher and the candidate teachers collaboratively conducted the study. The reason was to have immediate discursive views from participants who are required to be implementing the model after completion of the course (cf. Method). However, the topic for the lesson study was unanimously chosen by both the teachers and the candidate teacher, but was linked to larger national goals (National Curriculum Statement-NCS). The lesson study model focuses on one of the area, thus mathematics professional development of teachers. This is one version of an “inquiry group studies” as a way to improve mathematics teaching. In this specific study, as part of a goal to improve candidate teacher’s problem-solving, the study worked on a lesson study topic of subtraction with. Teachers, (intermediate phase), met weekly (block sessions) to design, teach and evaluate research lesson. The next step involved to revise the lesson, re-teach it, evaluate, reflect on the lesson again, and share results, this process took up to a year for each group involved in the study. Thus, the first group (experimental-2008 cohort) used LSM process for one academic year as part of the mathematics lectures. The 2009 cohort (controlled) used the normal lesson plan without applying any aspect of LSM.

Takahashi (2007) noted that lesson study empowers individual teachers and leads to steady incremental improvement in teaching, rather than fast reform which is often the unachieved goal of South African approaches to change. Takahashi (2007) summarize lesson study through an eight step problem-solving process, although others divide this process differently.

The next section of the paper describes the motivation for the propositions (hypotheses) in respect of the theory underlying the LSM.

Motivation for Propositions

Until recently, the LSM perspective has been promoted largely within disciplinary boundaries and in isolation from each other, though researchers have seriously addressed the scope for more integration (Takahashi 2007). Some research (Kajander et al. 2005) showed that mathematics lecturers do not use the same definitions of content knowledge as well as knowledge of the process of pedagogy. When teaching, in particular mathematics, South African educators focus on a structured lesson plan, whereas the Japanese concern themselves with LSM which incorporated a number of additional such (a) brief description of classroom context (b) materials/resources, (c) instructional objectives (d) introductory activities (e) instructional strategies/student activities (f) closure (g) assessment (h) duration and (i) alignment/consistency, noting that most of these are components of a lesson plan.

But LSM have been very influential not only in Japan but also in the USA. The reason being that the model has made an important contribution to the understanding of mathematics teaching across the Asian pacific and the West (Chokshi and Fernandez 2004; Fernandez and Yoshida 2004). Thus, the hypothesis is that the
most efficient and effective student learning of mathematics would result when classroom instruction and materials are aligned with lesson study model. Following the above hypothesis, research on curriculum alignment in Japan and the US lately tends to favor LSM as a positive influence on achievement of Mathematics (Fernandez and Yoshida 2004).

When the literature was examined in SA, there were a limited number of publications about the LSM, although there were several publications about the curriculum studies. While, a few of these curriculum studies are research (Reddy et al. 2007), most of them are about the description and discussions about the curriculum development, teacher education and multigrade teaching. Although, the introduction of the LSM to Japanese education was quite parallel to its development and application in the United States, it is taking some time for it to be implemented in practice and in theory in South Africa.

Following the lack of attention given to LSM in South African schools, this study used methodologies derived from LSM to study the effective teaching of mathematics in a historically Black South African University students pursuing a Postgraduate Certificate of Education (PGCE) Programme. The paper presents empirical work related to the scholarly search for effective teaching of mathematics in order to determine major issues of importance for future research and to understand the issues in relation to theory and application of LSM in South Africa. This study could therefore contribute to the South African literature on mathematics education, since there are few experimental studies related to the LSM. Moreover, the findings of this research might start to guide the attempts to develop teaching at pre/in-service teacher education by using the LSM.

**Research Hypotheses**

Following the assertion that application of LSM improves effective teaching and learning of mathematics. The following have been hypothesised:

**Hypothesis 1**

Ho= There is no significant difference in using LSM in the teaching of mathematics

Ha= There is significant difference in using LSM in the teaching of mathematics

**Hypothesis 2**

Ho = LSM is not a better predictor of improving mathematics teaching

Ha= LSM is a better predictor of improving mathematics teaching

**Hypothesis 3**

Ho= Usage of LSM by teachers is not evaluated differently by different teachers

Ha = Usage of LSM by teachers is evaluated differently by different teachers

**Hypothesis 4**

Ho = Distinct views on LSM could not be identified by teachers

Ha = Distinct views on LSM could be identified by teachers

**METHODOLOGY**

The purpose of this two-phased sequential mixed methods (Creswell 2003) study was to obtain statistical results (phase I) from a sample (43) of Post Graduate Certificate of Education (PGCE) students over a period of two years. In the second phase, interviews were done in order to explore different aspects of planning via the LSM. The sample consisted of 43 cohorts of the PGCE students in a mathematics class (412E and 422E) over four semesters in University of Fort Hare in the greater Eastern Cape Province of South Africa. Thus, the sample was made of candidate teachers preparing to teach the intermediate phase (4-6 grades) of the South Africa schooling system. Suggesting that a replication of this study should take cognisance of the cohort of students in terms of (a) duration of programme and (b) purpose of the programme prescribed by the South African National Curriculum Statement. This was because PGCE programme was designed for graduates of programmes other than a degree in the field of education.

These were then followed up with few respondents (13) to explore those results in depth. Thus, in the first phase, inferential research hypotheses (cf. research hypotheses) was con-
constructed in order to compare the effects of using the LSM constructs on a teachers ability to improve effective mathematics teaching.

In this study, the experimental group was the 2008 cohort (22 candidates), while the 2009 (21 candidates) were the controlled group. In the experiential group, the candidates were exposed to teaching using LSM, while the controlled were those using the lesson plan (LP) method. The mean scores of exams of the groups were compared by the researcher. It was hypothesised that there would be significant difference between those candidate teachers using the LSM and those using LP. Another recommendation thereof is that candidate researchers may use the same cohort as experimental and controlled as opposed to that used in this study.

**Procedure of LSM**

After getting information about the LSM to apply to the experimental group, next, the features that incorporated LSM and LP were used. Thus examples in the literature were searched and examined. A set of criteria that was used to assess teaching of mathematics was determined. The LSM characteristics had nine components: (a) brief description of classroom context, (b) materials/resources (c) instructional objectives (d) introductory activities (e) instructional strategies/student activities (f) closure (g) assessment/re-evaluation (h) duration and (i) alignment/consistency were explained in the inferential analysis.

Thus, after giving some exercises containing fifteen (15) questions, candidate teachers were asked to match objectives and the cognitive categories. They were also asked to write at least one question related to every sub-category in the cognitive process dimension. Then, the knowledge categories were explained, they were asked to give two factual, conceptual, and procedural knowledge examples from their topic area. Since, metacognitive knowledge was new for them, the instruction continued at a slow pace and every single sub-category was explained thoroughly. After explanations and discussions about these sub-categories, various activities and discussion were undertaken.

In the control group, the course was instructed in a traditional way with the methods of lecture, question-answer, and discussion and candidate teachers were given traditional information about how to prepare lesson plans to teach. In this regard, the research (lecturer) first asked them to examine lesson plans and gave them feedback about the lesson plans they had prepared.

**Details of Sampling**

The study assigned one of the year groups (2008 cohort) as the experimental group (n = 22) and the other (2009) as the control group (n = 21). The mean ages of the PGCE candidate teachers in the experimental and the control group were 21 years and 22 years respectively. Ethnically, while 4% of them made up Whites, the rest (96%) was Black students (Coloured and Black Africans). The analysis of repeated MANOVA and univariate repeated ANOVA were done to investigate whether there was a significant difference or not between groups in respect to components in the LSM. In order to examine the lesson of the experimental group frequency and percentage were calculated. In order to test inter-rater reliability of the scores obtained from the experts who assessed the lesson plans, ANOVA test, which was done had intraclass correlation coefficient based on Spearman-Brown formula as 0.94, suggest a reasonably strong reliability.

**RESULTS AND DISCUSSION**

This section addresses the results of the study in light of the hypotheses posed. It consists of six sub-sections. The first includes test of assumptions of repeated-measures ANOVA and MANOVA (Tabachnick et al. 2001; Hosmer and Lemeshow 2000) and preliminary analysis of the groups. Although, the test of Mauchly’s sphericity was as explained it is imperative to note that subsequent test have been conducted and explained as the entire sections developed.

**Test of Assumptions: Mauchly’s Sphericity**

Mauchly’s sphericity test for first hypothesis was conducted to examine the form of the common covariance matrix. The results revealed that sphericity assumption was not violated (X² = 32.5; df 3; p=0.45). Thus, the chi-square approximation for this test was 32.5 with 3df and an associated probability greater than 0.05. Since, this was greater than the alpha level of
0.05, the study was confident that the data did meet the sphericity assumption.

**Preliminary Analysis**

Firstly, a preliminary analysis to determine if there was any statistically significant difference between two groups was conducted in terms of their performance in their exams (cf. procedure of LSM). The results revealed that, the mean scores of the experimental group \( \text{M}=238.10, \text{SD} = 7.00 \) and that of control group is \( 238.18, \text{SD} = 9.90 \) was not significant. Thus findings of the independent sample t-test \[ t(46) = 0.06, p>0.05 \] was not statistically significant. The results suggested that the candidate teachers in the experimental and control groups are similar. Secondly, the researcher administered a pretest of instructional planning and evaluation course content via LSM to the groups to determine comparable levels of understanding of the content prior to the experiment. Noting that KR-20 reliability coefficient of the pretest was 0.86. The scores that were obtained from the pretest were examined by using independent samples t-test in order to determine if there was any statistically significant difference between the two groups. Again the results suggested that the mean of the pretest scores of the experimental group \( \text{M}=39.6, \text{SD} = 5.34 \) and that of control group \( 40.24, \text{SD} = 6.00 \) was insignificant. The t-test, which was done with the means of the pretest scores \[ t(46) = 0.56, p>0.05 \] was not statistically significant. Suggesting that the experimental and control groups are not different in respect to the pretest that was applied at the beginning of the semester of each year.

**Hypothesis 1**

This section sought to examine the null hypothesis that there was no significant difference in using LSM in the teaching of mathematics. A comparison of planning skills of the groups was conducted in order to investigate any significant difference in the LSM of the teachers in the control and experimental group in respect to components of LSM.

Multivariate analyses of variance (MANOVA) were done. The F ratio for MANOVA indicated that the differences between the two groups mean scores were statistically significant at the 0.05 level, \[ F (9, 44) = 5.62, p<0.05 \]. That is, the experimental and control groups had statistically significant mean scores on the collective dependent variables (cf procedure of LSM). The multivariate eta squared of 0.65 (based on Wilks lambda) implied that the magnitude of the difference between the groups was not small (Cohen 1988). That value indicated that 65% of multivariate variance of the dependent variables was associated with the treatment (LSM). Because a statistically significant MANOVA F was obtained for the collective dependent variables, univariate ANOVA was conducted to further understand how the two groups would be affected by the interventions regarding each of the dependent variables.

The results showed that a statistically significant mean difference existed between the groups with respect to closure, assessment, re-teaching and alignment/ consistency \( p<0.05 \). Additionally, a one-way-between-groups analysis of variance was conducted to explore the impact of age on levels of LSM. Subjects were divided into three groups according to their age (group 1:20 or less; Group 2:21 to 25; Group 3:26 and above). There was a statistically significant difference at the \( p<0.05 \) level for the three age groups \[ F (3,432) =4.53, p < 0.05 \]. Despite reaching statistical significance, the actual difference in mean scores between the groups was small. The effect size, calculated using eta square, was 0.2. Post-hoc comparisons using the Tukey HSD test indicated that the mean score for group 1 \( (\text{M}=20.14, \text{SD}=4.67) \) was significantly different from Group 3 \( (\text{M}=21.36, \text{SD}=4.35) \). Group 2 \( (\text{M}=21.26, \text{SD}=4.30) \) did not differ significantly from either group 3 as evidenced in the indexes.

Candidate teachers’ opinions about the planning with the LSM stated that the LSM had positive effects on their study of mathematics. While only two teachers pointed out that the LSM created some difficulties while teaching, others mentioned the contributions of the LSM: The researcher on the other hand notes that the LSM was very useful in planning even though using the LSM seems to be complex and difficult. A respondent

‘...while writing the objectives by using the LP was easier; I am having difficulty placing objectives by using the LSM.

When teachers were asked the tasks they enjoyed or had most difficulty with the LSM, they stated that the most enjoyed task was to fill in
the LSM. Even though they pointed out that they had some difficulty in filling the LSM as part of the requirement, they stressed that they really enjoyed filling in the LSM collaboratively and they got excited as if they were curiously solving a puzzle. Buyambo (a respondent) capture this as saying:

*It was delightful and stimulating. Completing the LSM by thinking about and analysing our own knowledge each time was more amusing than making something by using memory. Placing every single objective into the LSM and writing activities and assessments for them were like solving a puzzle, I enjoyed that a lot.*

Later, participants come back to objectives and showed three elements all together, which enable them to think more deeply. The evidence suggested most difficult task in planning was to separate factual knowledge (fk) from conceptual knowledge (ck) and to understand metacognitive knowledge (mk). Metacognitive knowledge was a little harder to understand, since it was a new concept that was not taught in any other course before. The lecturer (researcher) stated that since they were studying in the field of mathematics education, lessons needed to contain objectives about applying procedural knowledge. Moreover, they pointed out that they (participants) rarely used those objectives containing metacognitive knowledge, since they were having some trouble understanding this type of knowledge.

When the teachers’ answers are examined, it can be said that all of them had difficulty in filling in the LSM. However, it could not be concluded this process had negative inferences, since this process was interesting and enjoyable at the same time. According to teachers, the most time consuming task was to determine the place of objectives in the dimension of knowledge and cognitive process.

Conclusively, as evidenced by both the inferential analysis together with the empirical opinions from the respondents, the study confidently rejects the null hypothesis and concludes that there was significant difference in using LSM in the teaching of mathematics. The next section examined the second hypothesis.

**Hypothesis 2**

This second sub-section addressed the null hypothesis that LSM was not a better predictor of improving mathematics teaching.

A one-way between-groups MANOVA was performed to investigate LSM differences in dependent variables. Three dependent variables were used: (a) brief description of classroom context-BDC (b) materials/resources-MR (c) instructional objectives IO. The independent variable was LSM. Thus, if there is a prediction between the dependent variables and LSM then LSM influence effective mathematics teaching. Preliminary assumption testing was conducted to check for normality, linearity, univariate and multivariate outliers, homogeneity of variance-covariance matrices, and multicollinearity, with no violations noted.

There was a statistically significant difference between experimental and control on the combined dependent variables: $F(3, 413) = 2.02$, $p=0.02$; Wilk’s Lambda = 0.87; partial eta squared = 0.3. When the results for the dependent variables were considered separately, the only difference to reach statistical significance, using a Bonferroni adjusted alpha level of 0.014, was BDC: $F(1, 421) = 6.51$, $p=0.004$, partial eta squared = 0.4. An inspection of the mean scores indicated that experimental reported higher levels of BDC ($M=34$, $SD=7.9$) than control ($M=21$, $SD=5.2$).

Because the study was conducted with two different groups additional analysis was conducted to determine the difference with regards to time of the treatment (Intervention). A one-way repeated measures ANOVA was conducted to compare scores on the confidence in coping with mathematics test at Time 1 (prior to the intervention) ($M=17.00$, $SD=4.21$), Time 2 (following the intervention) ($M=23.01$, $SD=4.42$) and Time 3 (three-month follow-up) (29.13, $SD=4.20$). The means and standard deviations suggest significant results. There was a significant effect for time [Wilk’s Lambda=0.33, $F (2, 24) = 38.10$, $p<0.05$, multivariate partial eta square = 0.85].

As evidenced above, the inferential analysis supports the alternate hypothesis while rejecting the null. The study therefore concludes that LSM is a better predictor of improving mathematics teaching. The next section examined the third hypothesis.

**Hypothesis 3**

The third hypothesis to be addressed was hypothesis 3. The section on hypothesis three
(3) suggested that the usage of LSM by teachers was not evaluated differently by different teachers. The first multivariate test of a within-subjects effect was the within-subjects main effect test. It examined changes in evaluation rate of LSM as a function of mathematics teaching. The null hypothesis was that the mean evaluation rate does change across respondents.

The results were significant, since the F ratio for this hypothesis was very large \( F(2, 141) = 2431.1, p = .0001 \), the study confidently rejected the null hypothesis and conclude that the evaluation rate changes in the population from which the sample was drawn.

**Hypothesis 4**

The main hypothesis (H0) tested was that distinct views on LSM could not be identified by teachers using LSM.

The mean view of the one-way repeated-measures ANOVA was 3 strongly support on the first test; 5 strongly support on the second test; and 14 strongly support on the third and final test (for experimental group-22 sampled). The ANOVA shows that these views are significantly different, \( F(2, 17) = 32.11, p<0.0005 \). Repeated-measures t-tests showed that subjects were significantly slower on the first test than they were on the second (test 1 versus test 2: \( t (6) = 5.32, p<0.001 \); but there was further increase in completion mean-view between the second and third tests (test 2 versus test 3: \( t (6) = 5.51, p = .001 \), significant).

It appears that practice produces an initial rapid improvement in subjects’ view of performing a task, slows it, but then additional practice leads to increase and or further improvement.

Note that before the hypothesis 4 was conducted, an assumption was tested. The following section gave the brief results of a test to satisfy one of the requirements for doing repeated-measures ANOVA-called “Sphericity Assumption” in this particular case.

Recall, the sphericity assumption is that the variances of variables are equal – it is the equivalent of the homogeneity of variance assumption applied in the “between subjects case” (cf. Tabachnick et al. 2001 for details), noting that if the test produces a significant result, the sphericity assumption has been violated. This means the p-value for the test of the within-subjects factor needs to be adjusted, thus the p associated with the Huyn-Feldt correction. In this particular case, the Mauchly Sphericity test was not significant (\( p = 0.169 \), which is greater than .05), so no violation of assumption.

As seen from Table 1, the sphericity assumption was satisfied for these data. In this case, there was a highly significant effect of the “meanview” variable; in other words, there is a significant difference between the three tests in terms of the average views taken to complete the task (\( p<0.0005 \)).

According to the responses, although they found LSM a little confusing at the beginning, they started to enjoy preparing their teaching (re-teach) because of the examples given at the course and the discussions. This, as a respondent (Job) captured, stated that:

*When I first started LSM, I was afraid of being unsuccessful since it seemed confusing. However, I understood that the task was not that difficult. Moreover, I realised that preparing LSM would not take much effort, since determining activities and assessments were much easier after completing the objectives. Preparing LSM gave me opportunity. Thus, to prepare more detailed, re-teach and to think deeply. I believe that I can prepare clearer, more understandable, and more valid.*

On the other hand and consistent with the respondents, participants opinions about planning with the LSM was conclusive on one thing. The participants pointed out that teachers were curious and interested in preparing LSM. The study observed high motivation towards making teaching suggesting that this motivation was

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\( a = \) may be used to adjust the degrees of freedom for the average tests of significance. Corrected are displayed in the tests of within-subjects effects table, which was explained.
caused by the structure of the LSM since they (respondent) knew that they were studying something model, which was not observed in the controlled group.

In general, the findings of the study suggest that planning with the LSM is effective and joyful even though it is more time consuming and requires more effort. It could be said that both researcher and respondents had a positive attitude towards planning mathematics teaching with the LSM.

CONCLUSION

Based on the analyses in section three (3) together with the empirical evidence from respondents, the results have been consistent with the success of previous writers.

Two main findings were distinct and emerged, thus: (a) LSM would be a better predictor of improving mathematics teaching and (b) distinct views on LSM could be identified by the mathematics teachers in the process of using LSM. The results of this study suggested that it is harmonious with and confirms those studies discussing potential benefits of planning of mathematics teaching with the LSM. It could be said that there would be several improvements in mathematics curricular development by the application of the LSM in South African mathematics education.

RECOMMENDATIONS

The implication from the study was that firstly; the LSM could be accepted as a turning point in developing the metacognitive skills, emphasising the reflective teaching and learning, and providing internal consistency of instructional planning. The LSM may provide a framework within which prospective teachers as well as teachers could model not only the way they teach, but also the way they examine and analyse their teaching.

NOTES

1 For practical purposes, this is important only in helping one to decide which output to use, and if the output should be adjusted. If one can use the univariate output, one may have more power to reject the null hypothesis in favour of the alternative hypothesis. However, the univariate approach is appropriate only when the sphericity assumption is not violated. If the sphericity assumption is violated (where p<0.05), then in most situations its better off staying with the multivariate output.

2 Names are pseudonym

3 Pen name

4 Most calculation report a separate multivariate test statistic (Pillais’, Hotelling’s, Wilks’, and Roy’s); but the Wilk’s test is commonly used.

REFERENCES


